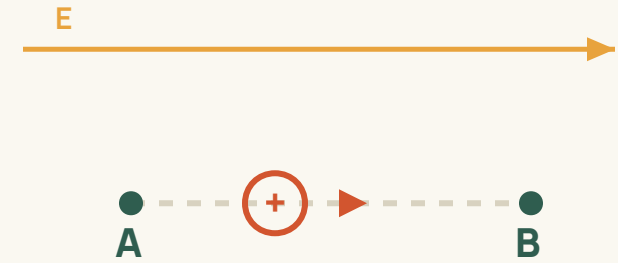


Voltage & Equipotentials

The "electrical height" between two points — and the surfaces that share it.

ESSENTIAL QUESTION

Moving from one point to another, why does each unit of charge gain the same energy — no matter which charge we carry?



two points, one voltage between them

From a familiar hill to a map of the field.

01

ANALOGY

Potential plays the role of height in a field.

02

DEFINE

Voltage is the difference of potential between two points.

03

WORK

The work on a charge is $W = qU$.

04

SURFACES

Equal-potential surfaces map the whole field.

ESSENTIAL QUESTION

What is the "voltage" between two points, and why is it set by the field — not by the charge we test it with?

• RECALL

Height is to gravity what potential is to a field.

You already know how a hill stores energy. The electric field works the same way.

GRAVITY



Height decides the stored energy. Work done by gravity:

$$W_{AB} = E_{pA} - E_{pB}$$

ELECTRIC FIELD



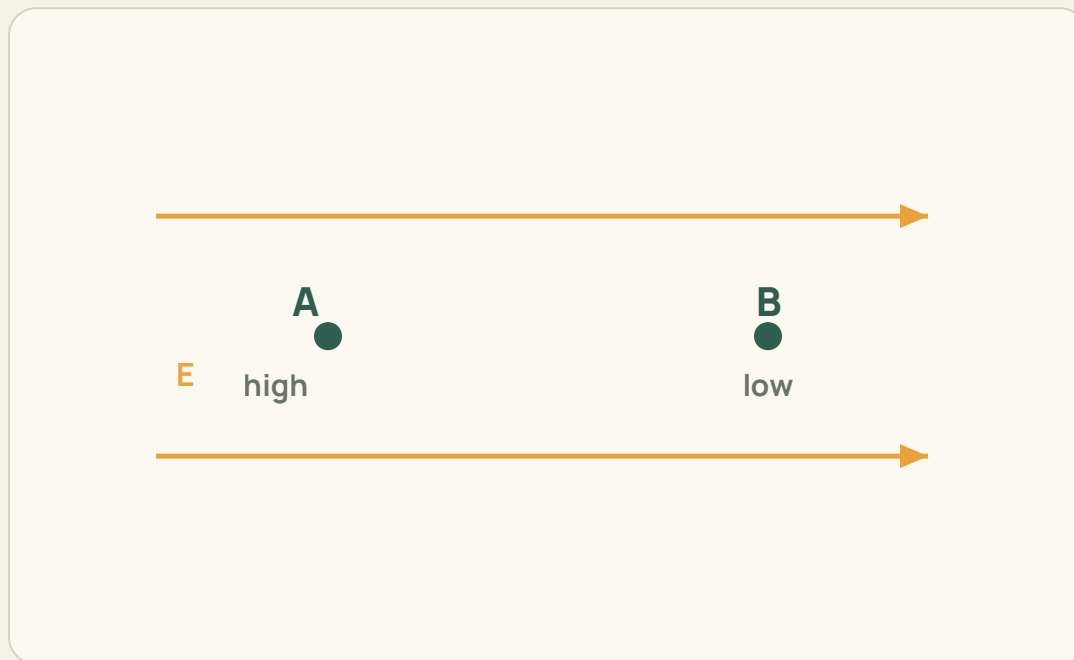
Potential φ decides the stored energy. Work done by the field:

$$W_{AB} = E_{pA} - E_{pB}$$

DEFINE

Voltage is the difference of potential between two points.

Pick two points A and B. Subtract their potentials — that difference is the voltage.



$$U_{AB} = \varphi_A - \varphi_B$$

φ_A potential at A
energy per unit charge there

φ_B potential at B
energy per unit charge there

U_{AB} the voltage from A to B
measured in volts (V)

MODEL

Voltage is a signed scalar – read its sign carefully.

Four facts that follow straight from the definition $U_{AB} = \varphi_A - \varphi_B$.

01

SIGNED

$U_{AB} > 0$ means A is at higher potential than B;
 $U_{AB} < 0$ means lower.

02

REVERSIBLE

Swap the order and the sign flips: $U_{AB} = -U_{BA}$.

03

NO DIRECTION

A scalar, not a vector. The sign shows relative height, not a direction in space.

04

REFERENCE-FREE

Shift where $\varphi = 0$ and every φ shifts together — the difference U_{AB} is unchanged.

Always name both points: U_{AB} and U_{BA} are different statements.

DERIVE

The work on a charge is its charge times the voltage.

Start from energy, swap in $E_p = q\varphi$, and the test charge factors out.

$$W_{AB} = E_{pA} - E_{pB}$$

$$W_{AB} = q\varphi_A - q\varphi_B$$

$$W_{AB} = q(\varphi_A - \varphi_B)$$

$$W_{AB} = qU_{AB}$$

WORKS IN ANY FIELD

No "uniform field" assumption was used.

$W_{AB} = qU_{AB}$ holds for every electrostatic field
— uniform or not.

CHECK

Voltage is fixed by the field, not by the charge you test it with.

A common trap: "since $U = W/q$, doesn't a bigger charge mean a bigger voltage?"

TEST WITH q

Between A and B the field does work W .

$$U_{AB} = \frac{W}{q}$$

TEST WITH $2q$

Double the charge and the field does double the work, $2W$.

$$U_{AB} = \frac{2W}{2q} = \frac{W}{q}$$

The ratio is the same. Voltage is decided by the source charges and the two positions — full stop.

APPLY

Four ways to find the work done on a charge.

Pick the one that fits what the problem gives you.

01

DEFINITION

Force along the path:
 $W = qEd.$
uniform field only

02

VOLTAGE

From the potential drop:
 $W_{AB} = qU_{AB}.$

03

ENERGY CHANGE

From stored energy:
 $W_{AB} = E_{pA} - E_{pB}.$

04

WORK-ENERGY

With other forces:
 $W_{elec} + W_{other} = \Delta E_k.$

Methods 02–04 work in any field; only 01 needs a uniform field.

MEASURE

A handy energy unit falls straight out of $W = qU$.

Push one electron through 1 volt and see how much energy it gains.

$$\Delta E_k = qU = (1.6 \times 10^{-19} \text{ C})(1 \text{ V})$$

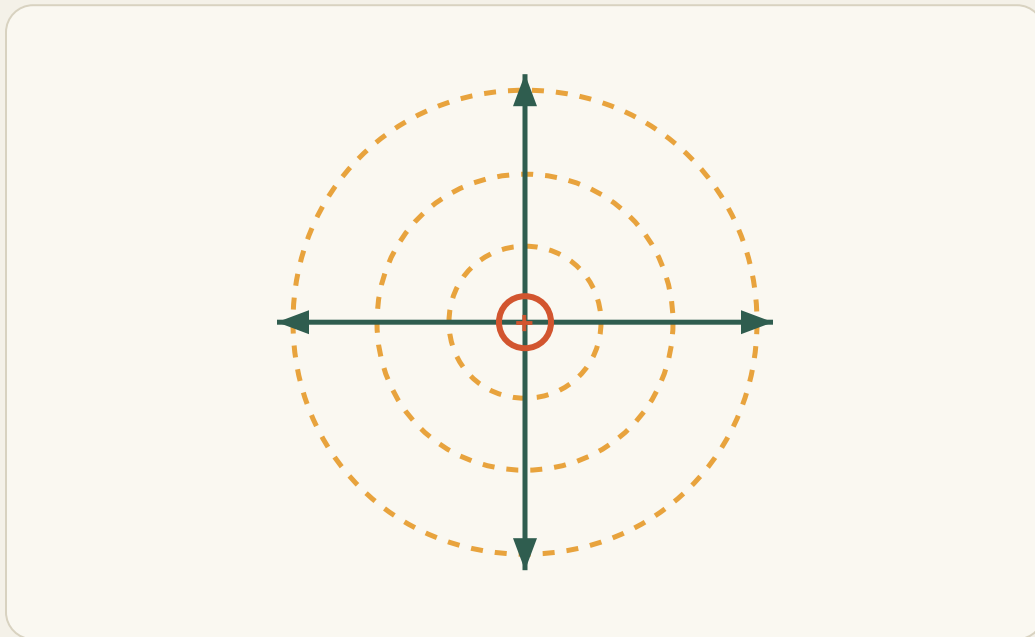
$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

One electron-volt is the energy an electron gains across a 1 V difference — the natural currency for tiny particles.

MODEL

Join all the points at one potential – you get an equipotential surface.

Like a contour line on a map: everywhere on it sits at the same "electrical height".



90°

Perpendicular to field lines
they always cross at a right angle

0

No work along the surface
 $W = 0$, since φ doesn't change

↓

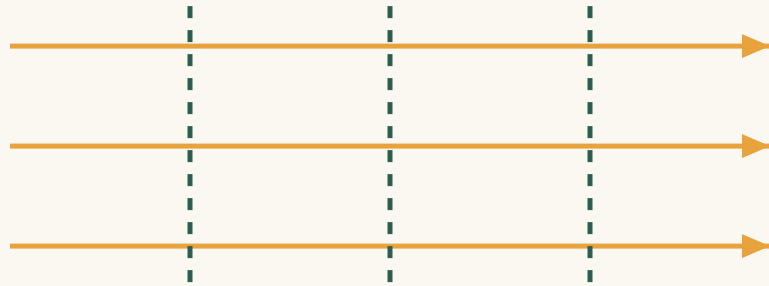
Field points high → low
from higher φ toward lower φ

COMPARE

Two fields, two families of equipotentials.

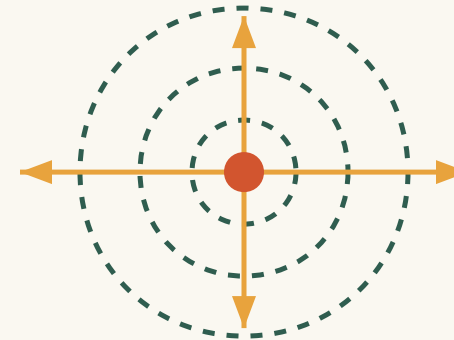
The shape of the surfaces tells you the shape of the field.

UNIFORM FIELD – FLAT PLANES



Evenly spaced parallel planes, square to the field.

POINT CHARGE – NESTED SPHERES

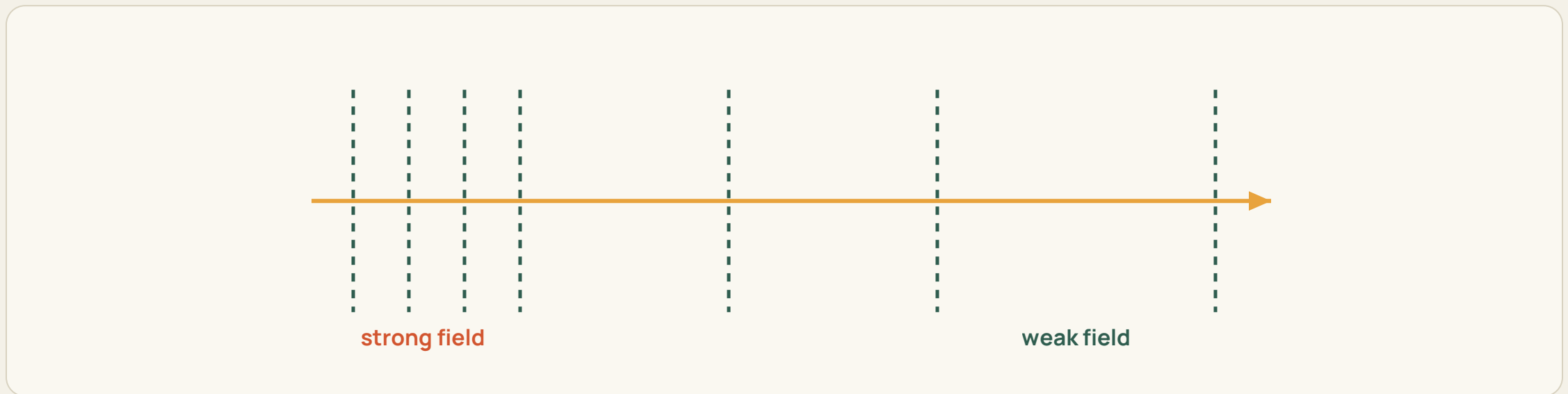


Spheres that spread apart as you move away.

OBSERVE

Crowded equipotentials mean a strong field.

Draw the surfaces at equal voltage steps; where they bunch up, the field is strongest.

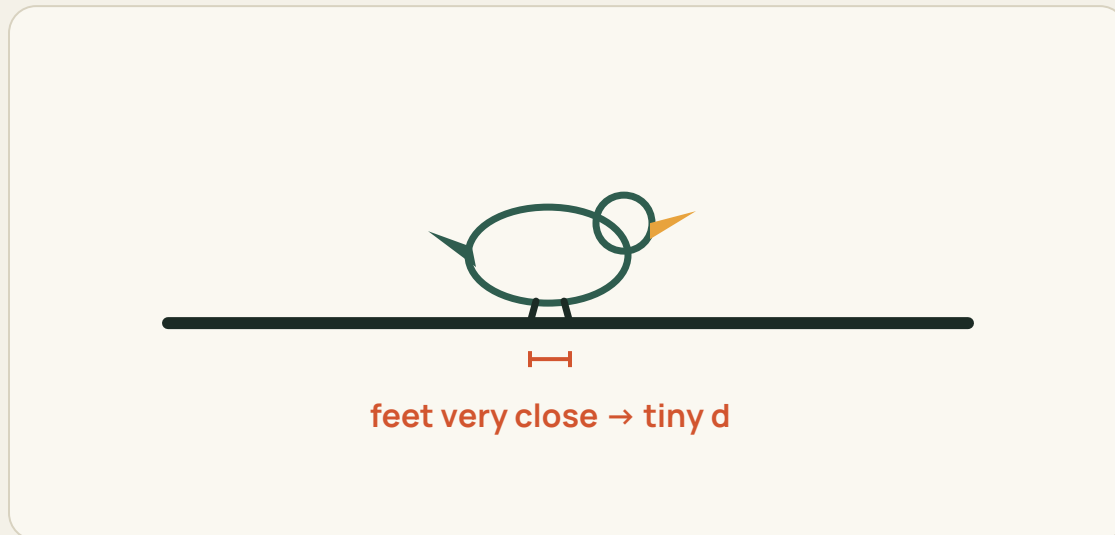


Equal voltage steps, smaller gaps \rightarrow larger E .

APPLY

Why the bird on a high-voltage line is safe.

The wire may be at hundreds of thousands of volts — yet the bird is unharmed.



THE REASON

Both feet touch nearly the same point, so the voltage between them is tiny. With a small U , the work $W = qU$ on charges in the bird is tiny too — no harmful current flows.

• SUMMARY

Voltage $U_{AB} = \varphi_A - \varphi_B$ is the field's "height drop" between two points. It sets the work on any charge through $W = qU$ – and the surfaces of equal potential map the whole field at a glance.