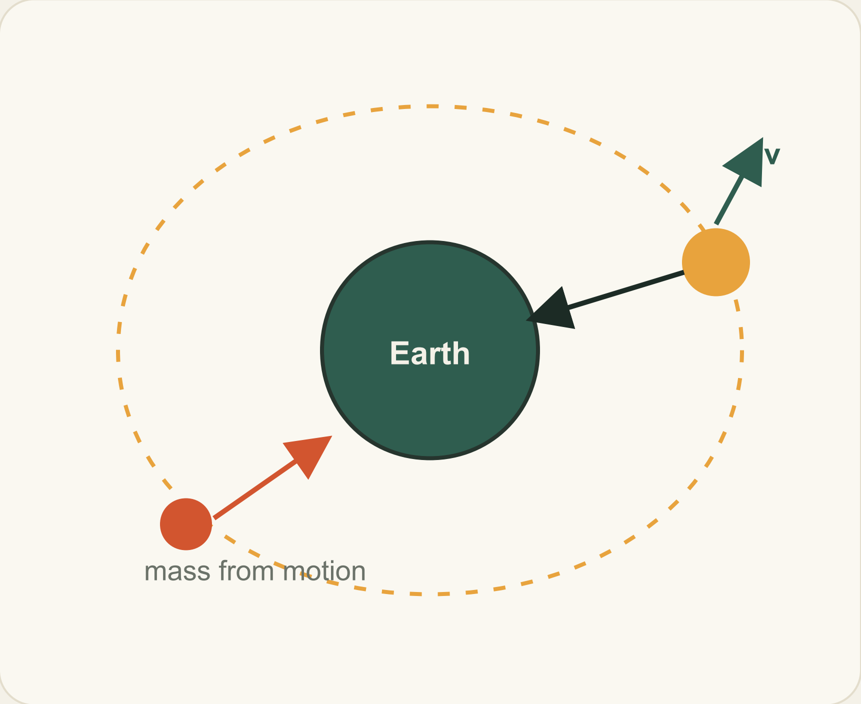
GRAVITY IN USE

Achievements of Gravitation.

Newton's law is not only a rule for forces. It became a tool for weighing worlds and finding bodies nobody had seen.

ESSENTIAL QUESTION

How can motion near one body reveal the mass, density, and even the existence of another?



LESSON MAP

Turn one law into four tools.

01

WEIGH EARTH

Use surface gravity once the gravitational constant is known.

02

WEIGH STARS

Use an orbiting body as a moving probe.

03

FIND DENSITY

Combine mass with radius, or use a low orbit.

04

PREDICT

Explain orbit errors and forecast returns.

THE COMMON IDEA

A body in circular motion acts like a measuring instrument: its orbit tells us the pull that guides it.

THE PROBLEM

A planet cannot be put on a scale.

A balance compares two objects placed on it. Earth is the object that makes the scale work, so ordinary weighing cannot weigh Earth itself.

A useful source image: ordinary weighing is local comparison.

1. **Direct weighing fails**
   The support, lever, or balance would still need something outside Earth to compare against.

2. **Gravity gives an indirect route**
   If we know the force law, nearby motion can reveal the mass creating the field.

3. **Cavendish supplied the missing number**
   Once the gravitational constant was measured in the laboratory, Earth's mass became calculable.

METHOD 1

Surface gravity weighs the central body.

1. **Near the surface**

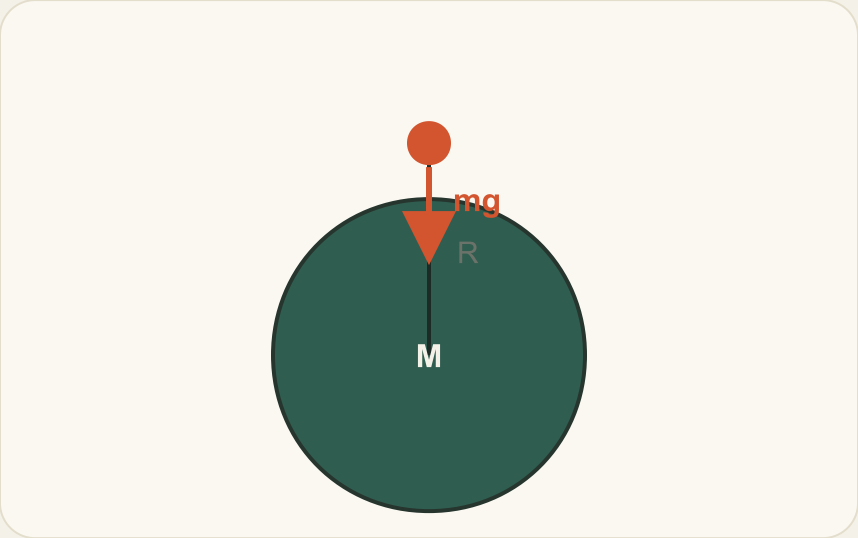
   A small test mass feels the weight force mg.

2. **The same force is gravity**

   $mg = G\frac{Mm}{R^2}$, where the unknown is the planet's mass.

3. **Cancel the test mass**

   The falling object helps us measure the planet, but its own mass disappears.



$$M = \frac{gR^2}{G}$$

WORKED RESULT

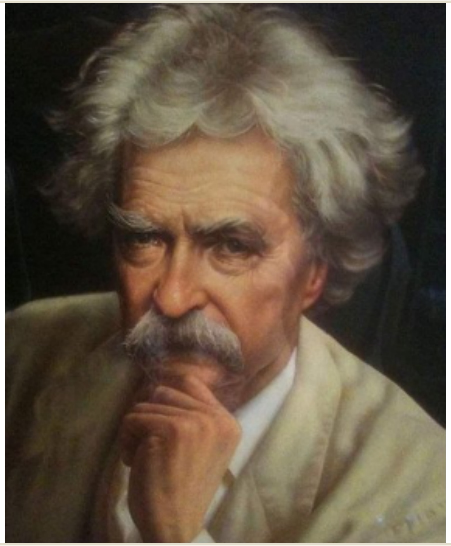
Cavendish made Earth's mass a number.

Use the measured values available near Earth: $g = 9.8\ \mathrm{m/s^2}$, $R = 6.4 \times 10^6\ \mathrm{m}$, and $G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$.

CALCULATION

$$M_E = \frac{(9.8)(6.4 \times 10^6)^2}{6.67 \times 10^{-11}}$$

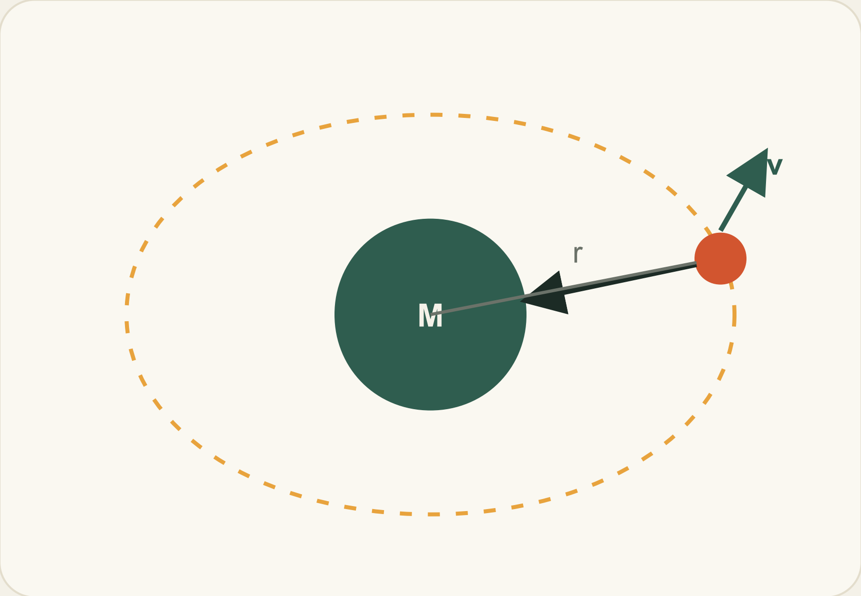
$$M_E \approx 6.0 \times 10^{24}\ \mathrm{kg}$$



Cavendish measured the gravitational constant, so the formula could be used.

An orbiting body weighs what it orbits.

For a near-circular orbit, gravity supplies the centripetal force. The orbiting mass cancels, so the method gives the central mass.



1. Start from the force balance

$$G\frac{Mm}{r^2} = m\frac{4\pi^2 r}{T^2}$$

2. Solve for the centre

$$M = \frac{4\pi^2 r^3}{GT^2}$$

3. Read the limitation

The orbit gives the mass of the body at the centre, not the mass of the satellite.

WORKED RESULT

Earth's year weighs the Sun.

Treat Earth's orbit as circular with $r = 1.5 \times 10^{11}\ \mathrm{m}$ and $T = 3.16 \times 10^{7}\ \mathrm{s}$.

ORBITAL CALCULATION

$$M_S = \frac{4\pi^2(1.5 \times 10^{11})^3}{(6.67 \times 10^{-11})(3.16 \times 10^{7})^2}$$

$$M_S \approx 2.0 \times 10^{30}\ \mathrm{kg}$$

The planet is the probe; the Sun is the unknown central mass.

Mass plus radius gives average density.

Density is not a new force idea. It is bookkeeping after a mass has been found.

From mass and radius

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3}$$

Using surface gravity

$$M = \frac{gR^2}{G} \quad \Rightarrow \quad \rho = \frac{3g}{4\pi G R}$$

Using an orbit

$$M = \frac{4\pi^2 r^3}{GT^2}$$

Near the surface

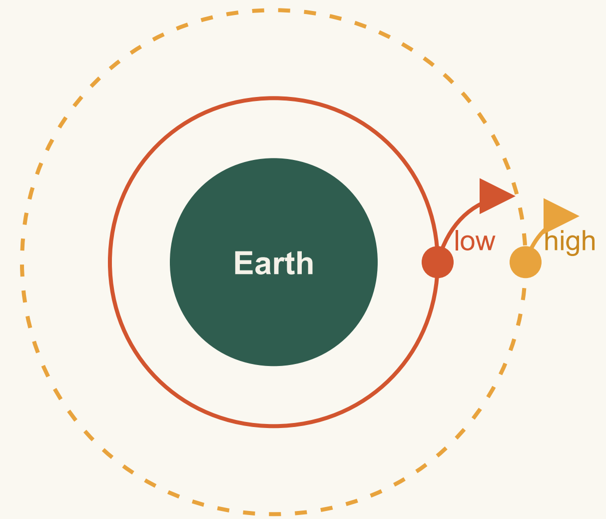
$$r \approx R \quad \Rightarrow \quad \rho = \frac{3\pi}{GT^2}$$

WHEN TO USE WHICH ONE

Use surface gravity and radius for a surface measurement; use orbital period and orbital radius when an orbiting body is available.

Higher circular orbits move more slowly.

For satellites around the same central body, the orbital radius controls every motion parameter.



Speed

$$v = \sqrt{\frac{GM}{r}}$$

Angular speed

$$\omega = \sqrt{\frac{GM}{r^3}}$$

Period

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Acceleration

$$a = \frac{GM}{r^2}$$

So a higher orbit has lower speed, lower angular speed, lower acceleration, and a longer period.

PREDICTION

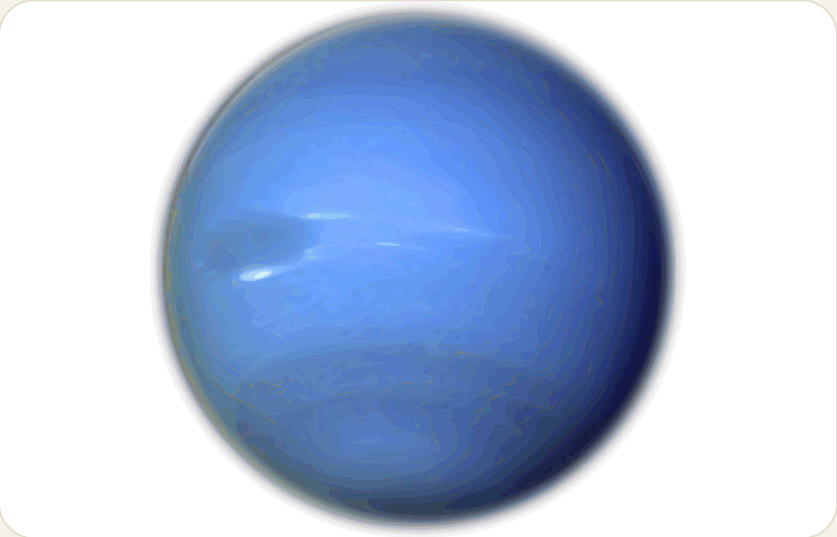
Orbit errors can reveal unseen worlds.

In the eighteenth and nineteenth centuries, Uranus did not quite follow the path computed from the known planets.

1. **Observation disagreed with calculation**
   Small residuals in Uranus's position suggested an extra gravitational pull.

2. **Mathematics pointed the telescope**
   Adams and Le Verrier independently estimated where another planet should be.

3. **Neptune was found in 1846**
   Galle observed it close to Le Verrier's predicted position - a major success of Newtonian gravity.



Neptune became famous as a planet found by calculation first.

PREDICTION

A comet's return can be forecast.

Halley recognised that several recorded comets were the same object moving on a long elliptical orbit.

1. **Gravity gives an orbit, not just a force**
   Once the path is known, the period can be estimated.
2. **Halley predicted a return**
   The comet returned in 1758-1759, after Halley's death, as his gravitational calculation implied.
3. **The next return is expected in 2061**
   The same law that weighs planets also makes long-term celestial prediction possible.

ACCURACY NOTE

Neptune is the clean historical triumph; Pluto's 1930 discovery followed a search but was not caused by a strong, accurate perturbation prediction.

SUMMARY

Gravity turned astronomy into measurement.

1. Surface method

   $M = \frac{gR^2}{G}$ weighs a body from surface gravity.

2. Orbital method

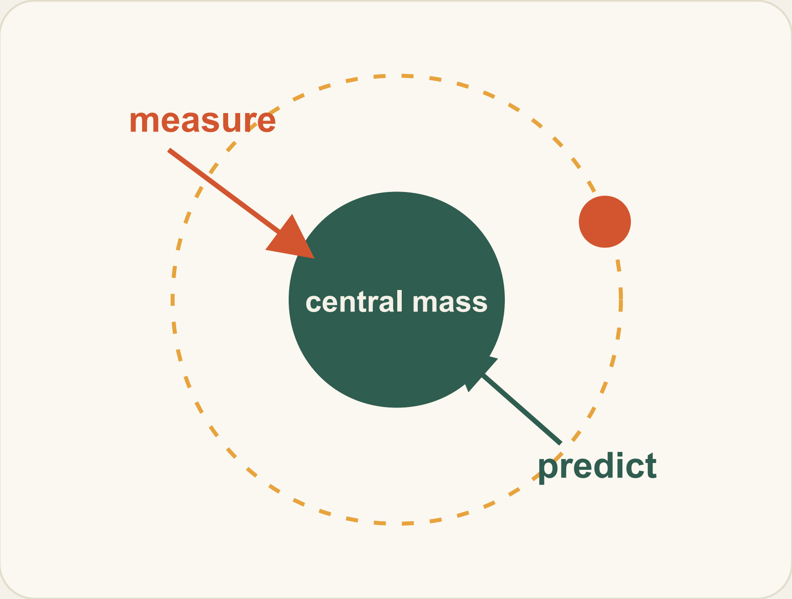
   $M = \frac{4\pi^2 r^3}{GT^2}$ weighs the central body from an orbit.

3. Density method

   $\rho = \frac{3M}{4\pi R^3}$, or $\rho = \frac{3\pi}{GT^2}$ for a near-surface orbit.

4. Predictive method

   Small deviations can point to unseen pulls, and known orbits can forecast future positions.



Motion is evidence for the force; the force is evidence for mass.