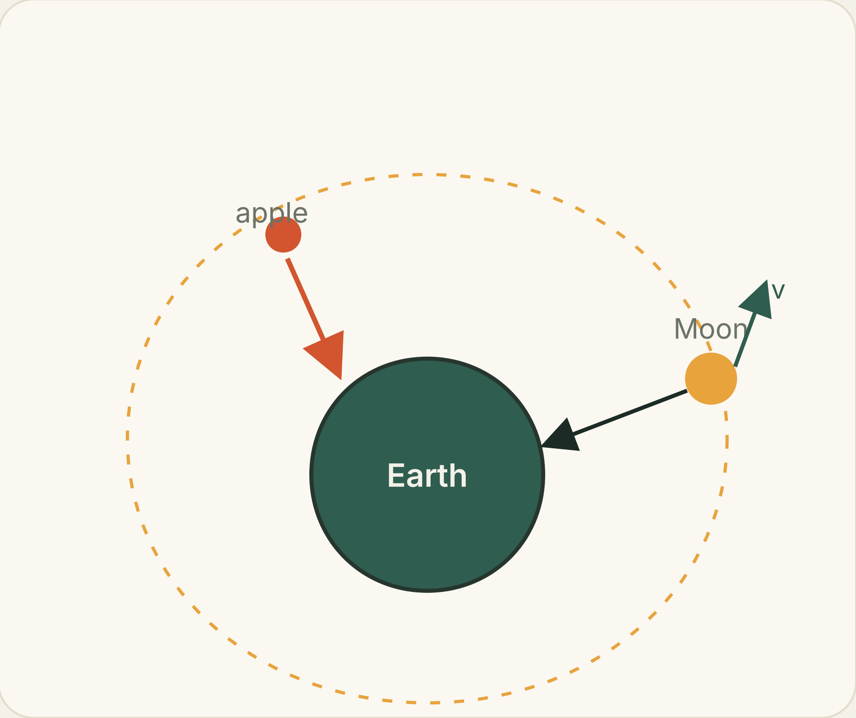
GRAVITATION

Universal Gravitation

The same pull that drops an apple keeps the Moon swinging around the Earth.

ESSENTIAL QUESTION

Why should one rule govern both a falling apple and the orbiting Moon — and how strong is that pull?



LESSON MAP

Build the law in four moves.

01

GUESS

Maybe the apple and the Moon feel the very same pull.

02

TEST

Does the pull weaken with distance? Check the Moon's fall against g.

03

LAW

Force grows with both masses and dies away as $1/r^2$.

04

USE

Measure G, then weigh planets and predict orbits.

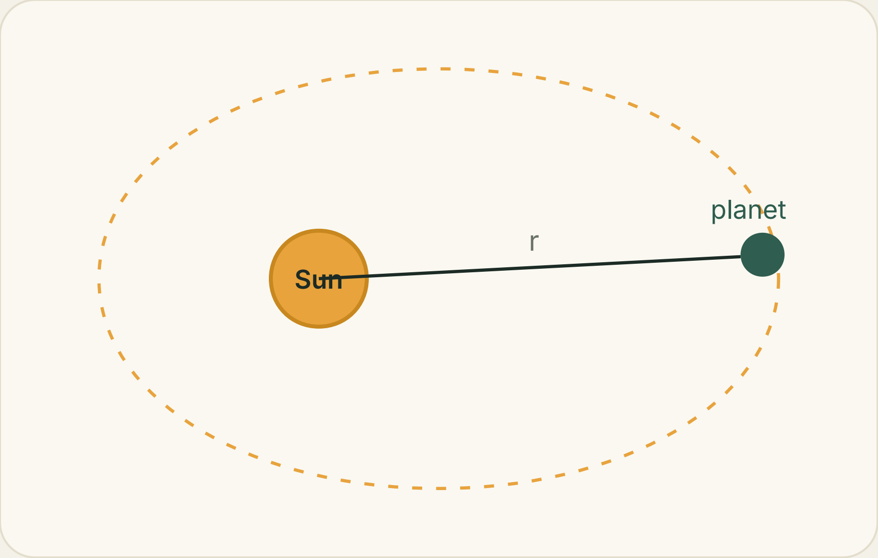
KEEP THIS IN MIND

Gravity is universal, mutual, and fades as the square of the distance.

RECALL

Kepler mapped the orbits — not the cause.

By 1619 the planets' motion was fully described. Yet nothing said why they keep turning.



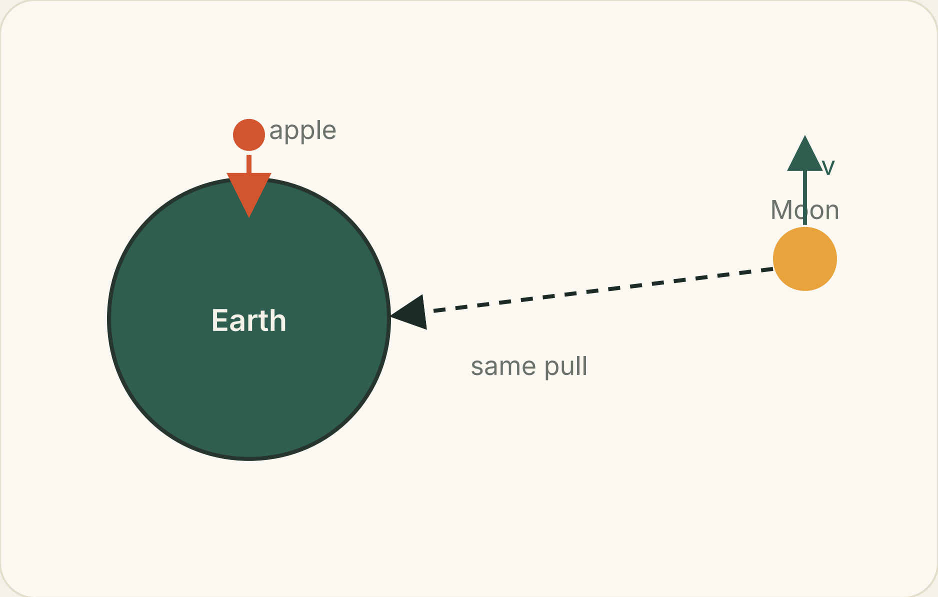
1. **Orbits are ellipses**
   with the Sun at one focus.
2. **Equal areas in equal times**
   a planet speeds up when it is closer.
3. **A period–size rule**
   $T^2 \propto r^3$ ties the year to the orbit.

These are the **what**. Newton went after the **why**.

THE BIG IDEA

One force reaches from the ground to the sky.

Newton's leap: the pull that bends the apple's path is the same pull that bends the Moon's.



The Moon is really **falling** toward Earth all the time — it just moves sideways fast enough to keep missing.

If one force does both jobs, it should weaken in a way we can **measure**. Let's check.

THE TEST

How fast does the Moon really fall?

The Moon sits about 60 Earth-radii away. Newton compared its fall to a stone's.

1. **Its centripetal pull**

   $$a = \frac{4\pi^2 r}{T^2} \approx 2.7 \times 10^{-3}\ \mathrm{m/s^2}$$

2. **Compare with surface gravity**

   $$\frac{a}{g} \approx \frac{2.7 \times 10^{-3}}{9.8} \approx \frac{1}{3600}$$

3. **And 3600 is no accident**

   $$3600 = 60^2 = \left(\frac{r}{R_E}\right)^2$$

THE VERDICT

Sixty times farther, and the pull is **3600 times weaker**. The force must fall off as

$$F \propto \frac{1}{r^2}$$

REASONING

What the force has to depend on.

1. **Both masses**
   The pull is mutual, so it must grow with each body: $F \propto m_1$ and $F \propto m_2$.
2. **The distance**
   The Moon test fixes the fall-off: $F \propto 1/r^2$.
3. **One constant to glue it**
   Fold the proportionality into a single number G.

$$F = G\,\frac{m_1 m_2}{r^2}$$

Put the three pieces together.

THE LAW

Newton’s Law of Universal Gravitation

$$F = G\,\frac{m_1 m_2}{r^2}$$

F	the gravitational pull on each body (newtons)
m_1, m_2	the two masses (kilograms)
r	distance between their centres (metres)
G	the gravitational constant

WHEN MAY I USE IT?

For two **point masses**, or for uniform **spheres** — then r runs centre to centre. Close, oddly-shaped bodies need calculus.

CHARACTER

Four promises the law makes.

01 · Universal

Every pair of masses attracts — apples, planets, galaxies, you.

02 · Mutual

Equal and opposite, by Newton's third law: $F_{12} = -F_{21}$.

03 · Inverse-square

Double the distance and the pull drops to a **quarter**.

04 · Along the line

It points straight from each mass toward the other — always attractive.

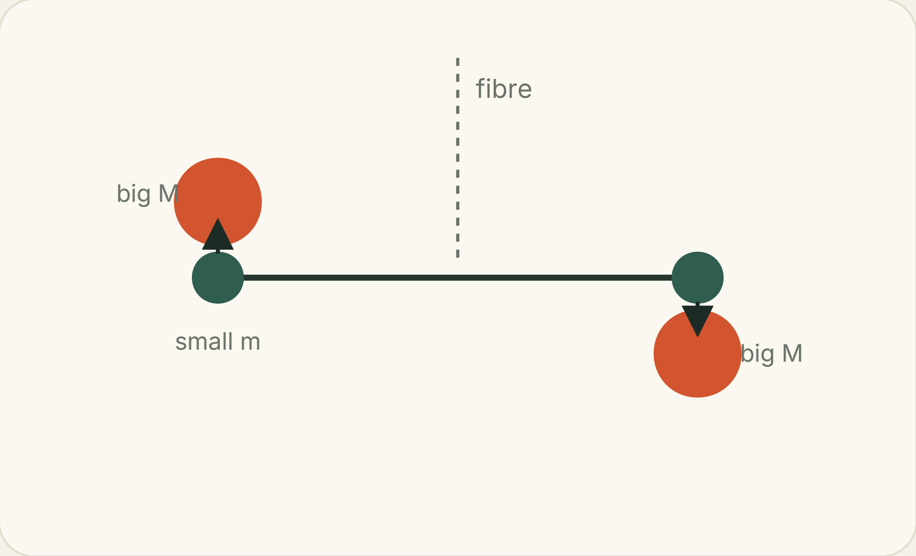
ONE PICTURE TO HOLD

$r \to 2r$ **means** $F \to \frac{1}{4}F$. **Distance matters far more than you'd guess.**

THE CONSTANT

Cavendish weighed the world.

A delicate torsion balance let him read off the tiny pull between lab masses — and so fix G.



$$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$$

So small that gravity is the **weakest** of the forces — only huge masses make it felt.

Once G is known, $g = \frac{GM}{R^2}$ hands you the Earth's mass.

WORKED EXAMPLE

Why you don’t feel the person beside you.

Two students, 60 kg each, standing 1 m apart. How hard do they pull?

1. **Drop the numbers in**
   $F = G\frac{m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11})(60)(60)}{1^2}$
2. **So the mutual pull is**
   $F \approx 2.4 \times 10^{-7}\ \mathrm{N}$ — about the weight of a speck of dust.
3. **Earth, meanwhile, pulls with**
   $mg = 60 \times 9.8 \approx 590\ \mathrm{N}$.

THE LESSON

The law always applies — but it takes a **planet-sized** mass before you ever notice the pull.

● SUMMARY

Every mass pulls every other with $F = G\frac{m_1 m_2}{r^2}$ — universal, mutual, and fading as $1/r^2$. Pin down G, and the whole solar system falls into place.