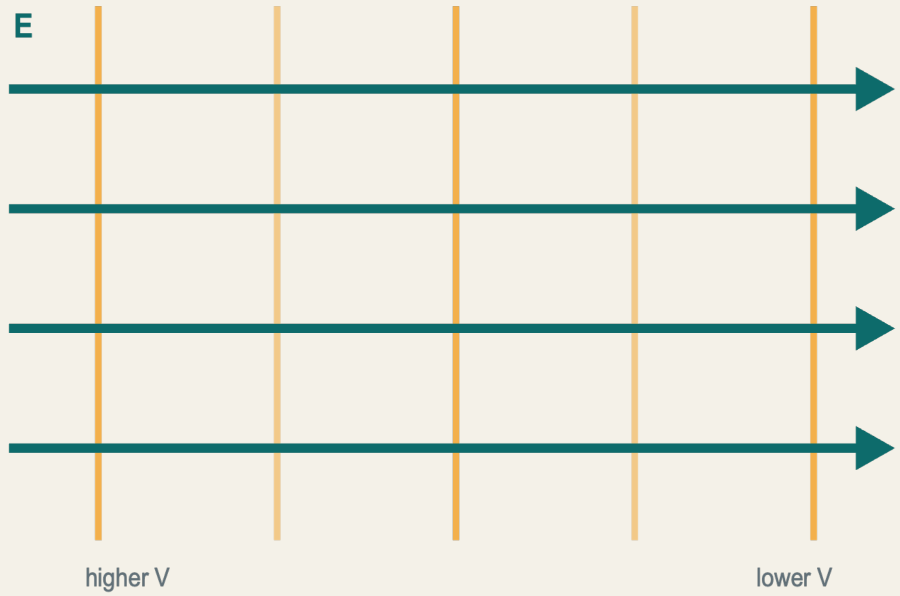
Potential Gradient

How voltage changes across an electric field

Essential question
Why are equipotential lines closer together in a stronger field?

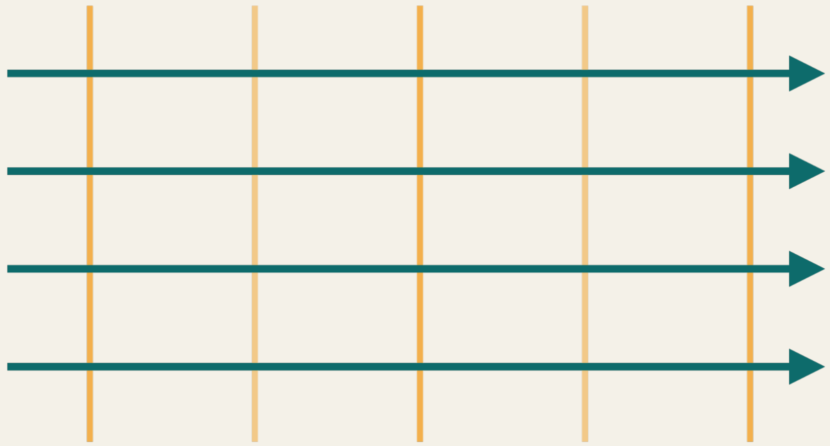


Potential falls in the direction of E

One potential drop, two field strengths

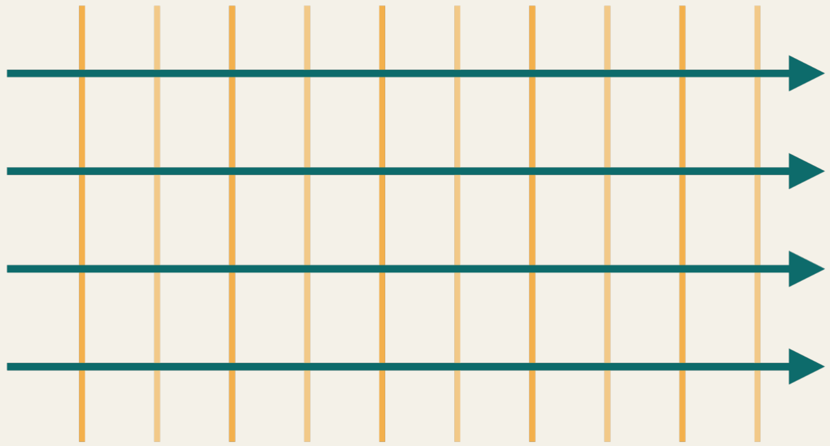
Equal-step equipotentials act like a visual ruler for the field.

WEAKER FIELD



Same ΔV over a longer distance

STRONGER FIELD



Same ΔV over a shorter distance

DERIVE

Calculate electric work in two ways

Uniform field; B is a distance d downfield from A.

FORCE × DISTANCE

$$W_{AB} = Fd$$

$$F = qE$$

$$W_{AB} = qEd$$

WORK PER UNIT CHARGE

$$U_{AB} = W_{AB} / q$$

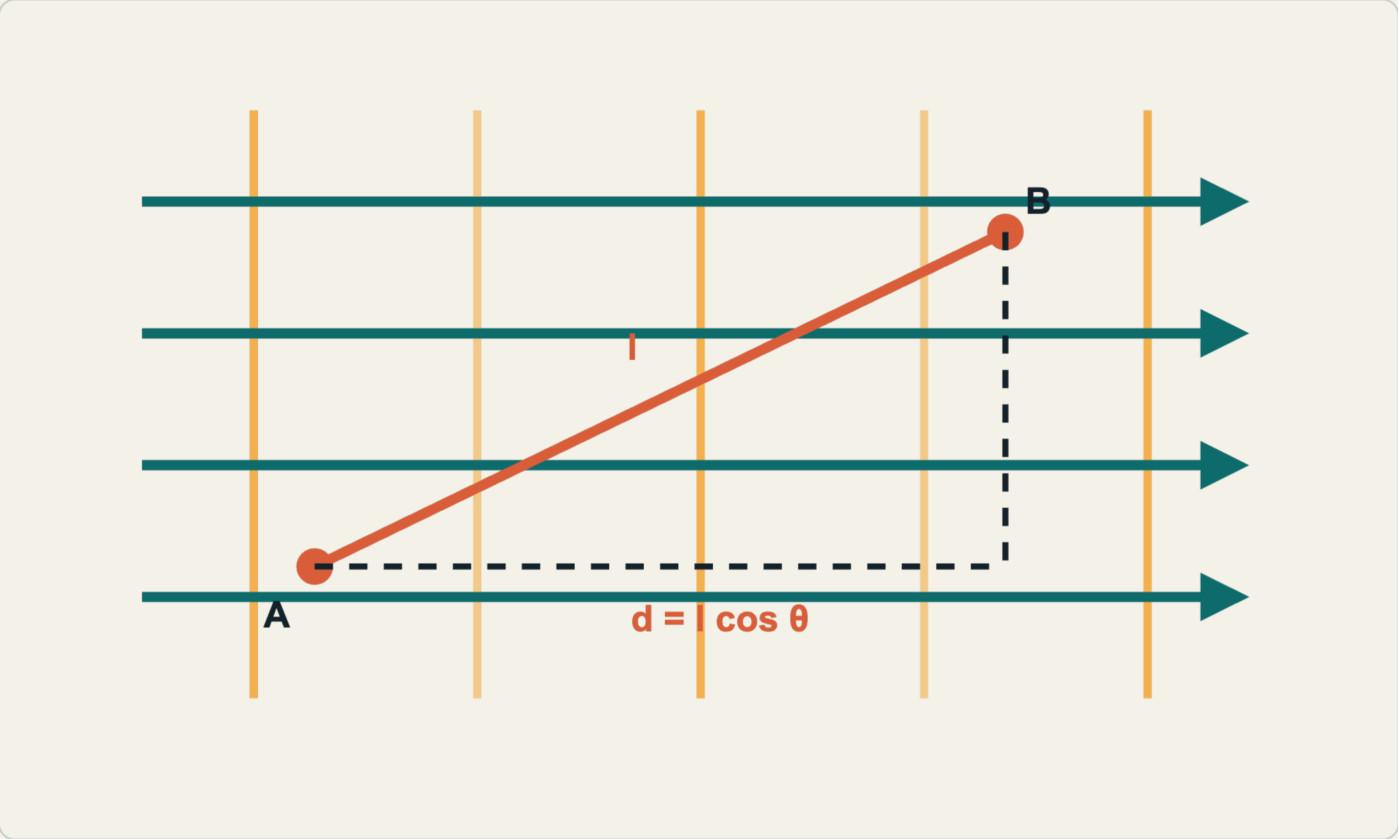
$$\text{so} \quad W_{AB} = qU_{AB}$$

$$U_{AB} = Ed$$

Convention: $U_{AB} = V_A - V_B$, so U_{AB} is positive when B is downfield from A.

The field only ‘sees’ the parallel component

For an oblique displacement, project the journey onto the field direction.



$$d = l \cos \theta$$

$$U_{AB} = E d$$

$$U_{AB} = E l \cos \theta$$

Perpendicular motion contributes zero.

COMPARE

Direction controls the potential change

Potential is a scalar, but displacement relative to E determines its change.

MOVE WITH E

→

V decreases

$\Delta V < 0$

MOVE ACROSS E

↑

V is unchanged

$\Delta V = 0$

MOVE AGAINST E

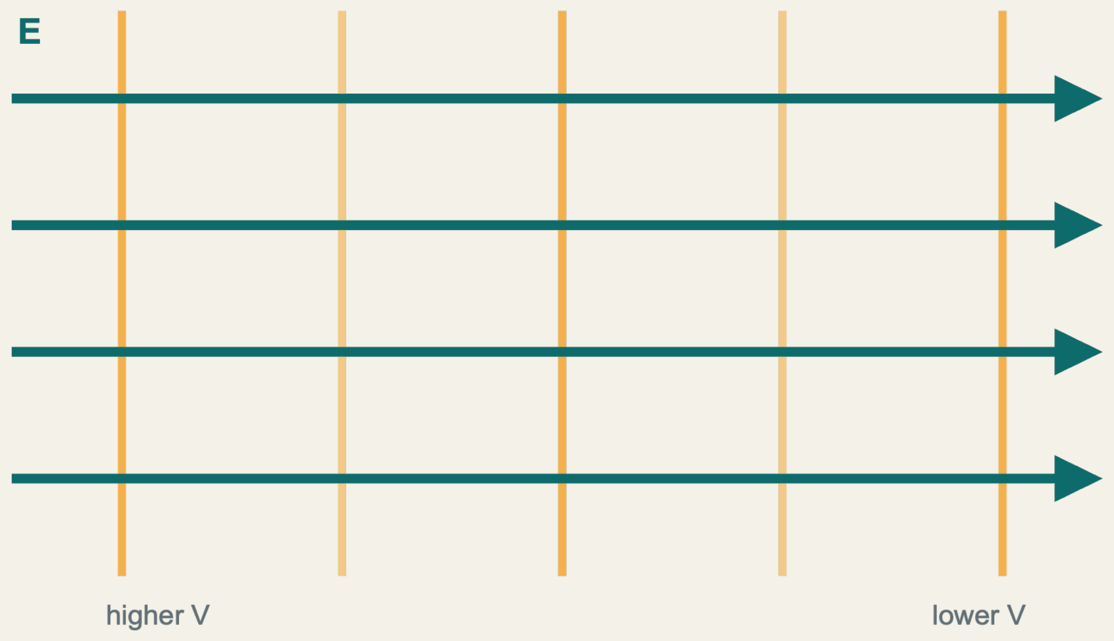
←

V increases

$\Delta V > 0$

Equipotentials are perpendicular to electric fields

Moving along one equipotential requires no electric work.



Along an equipotential

$$\Delta V = 0$$

$$W = q\Delta V = 0$$

The field points in the direction of the fastest potential decrease.

Field strength is potential drop per unit distance

This is the physical meaning of the potential gradient.

$$E = U / d$$

potential drop
over distance along E

UNIT CHECK

$$1\ \mathrm{V\,m^{-1}} = 1\ \mathrm{N\,C^{-1}}$$

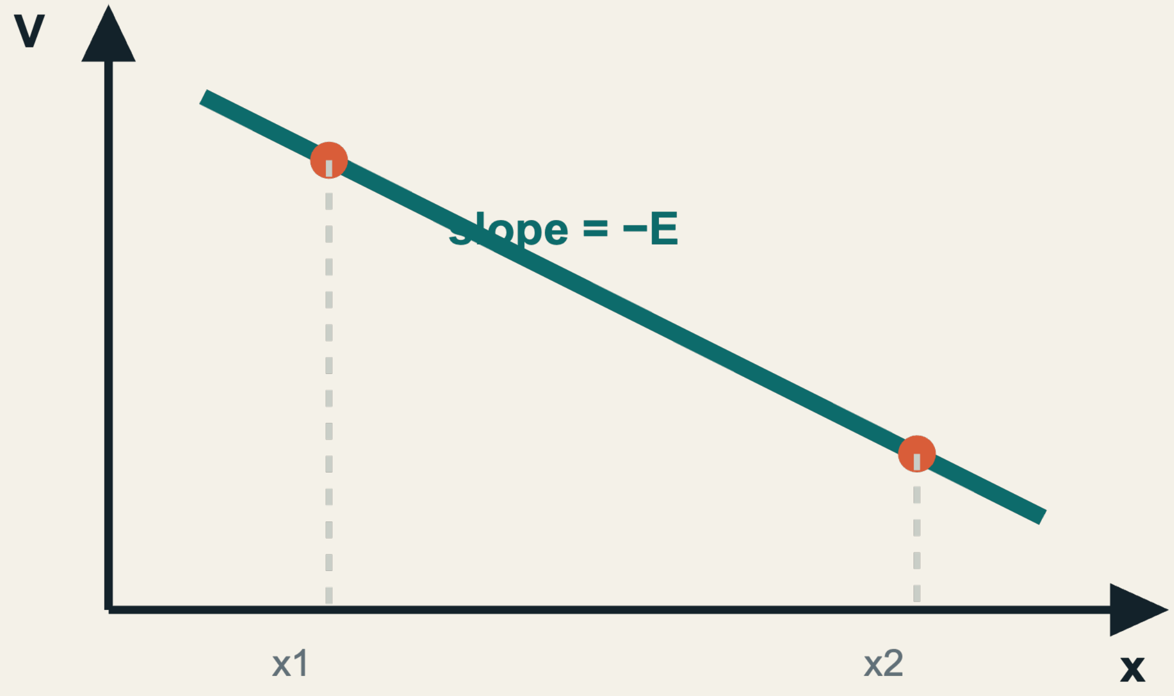
A field of 200 $\mathrm{V\,m^{-1}}$ means potential falls by 200 V for every metre travelled along E.

Larger E → faster potential change

GRAPH

A uniform field gives a constant potential gradient

Potential decreases linearly with position along the field.



$$\Delta V = -E\,\Delta x$$

$$dV / dx = -E$$

The minus sign says potential decreases in the direction of E.

EXPLAIN

Why strong fields have tightly spaced equipotentials

Compare neighbouring lines that differ by the same potential step.

For a fixed ΔV:

$$d = \Delta V / E$$

increasing field strength →

Small E

large spacing d

Potential changes slowly.

Large E

small spacing d

Potential changes rapidly.

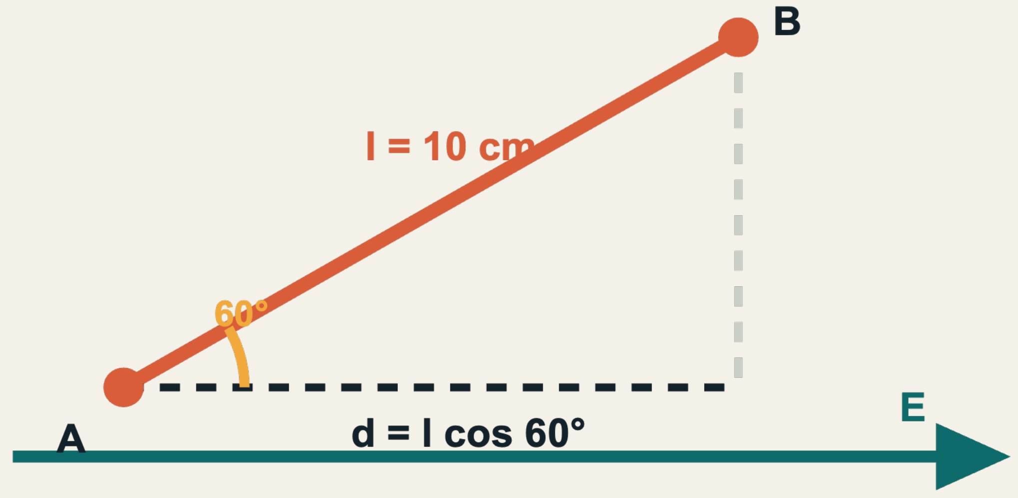
Three misconceptions to remove

Read the equation as a relationship within a field, not as a cause.

"Choosing a larger U creates a larger E."	No. E is a property of the existing field; U depends on which two points you choose.
"Any direction of decreasing potential is E."	No. E points toward the fastest decrease in potential.
"U = Ed works for every finite path."	No. The simple finite formula is exact only in a uniform field.

Worked example: use the projection

A uniform field has $E = 250\ N\ C^{-1}$. Find U_AB.



1 Project

$d = 0.10 \cos 60° = 0.050\ m$

2 Calculate

$U_{AB} = Ed$

$= 250 \times 0.050$

$= 12.5\ V$

TOOLKIT

Three ways to find a potential difference

Choose the method that matches the information available.

01

From potentials

$$U_{AB} = V_A - V_B$$

Use when both point potentials are known.

02

From work

$$U_{AB} = W_{AB} / q$$

Use when electric work and charge are known.

03

From a uniform field

$$U_{AB} = Ed$$

Use the projected downfield distance.

FINISH

Know the scope, then test your understanding

The core idea survives beyond uniform fields, but the calculation changes.

UNIFORM FIELD

$$U_{AB} = Ed$$

Exact for a finite separation when d is the downfield projection.

NON-UNIFORM FIELD

$$E = -\nabla V$$

Use the local gradient: field points toward the fastest potential decrease.

EXIT TICKET | Equal-step equipotentials become twice as close. What happens to E?