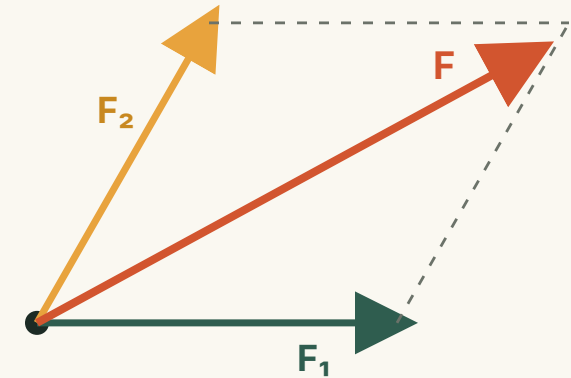


Composing and Resolving Forces

Two ropes can do the work of one. The trick is knowing how the pulls add — and they don't add like ordinary numbers.

ESSENTIAL QUESTION

When can a 3 N pull and a 4 N pull combine to give exactly 5 N — and never 7 N?



Four moves with force arrows.

01

REPLACE

One resultant can stand in for several forces — same effect.

02

COMBINE

At an angle, forces add by the parallelogram rule, not arithmetic.

03

RESOLVE

Run it backwards: split one force into two useful pieces.

04

VECTORS

Why all of this works — force is a vector, not a number.

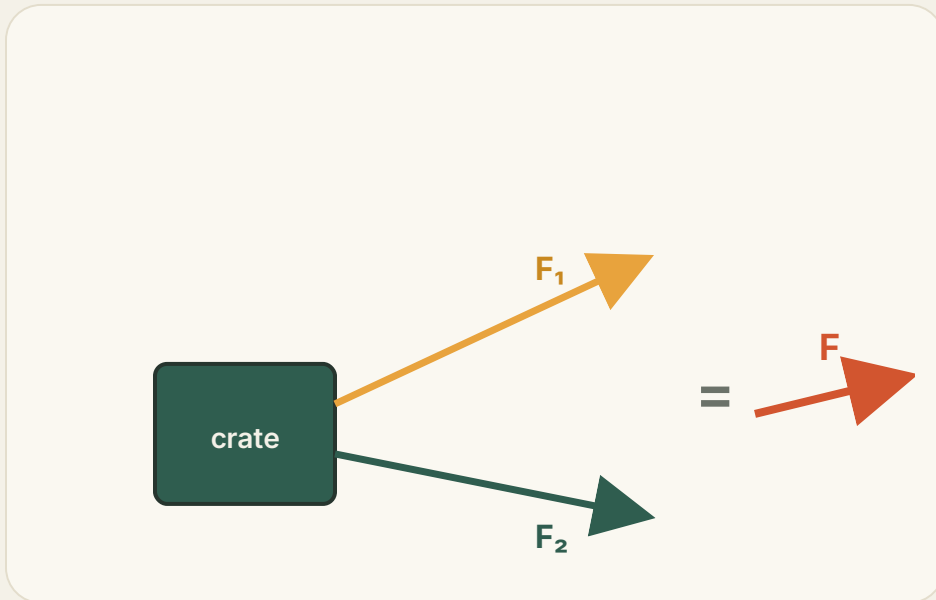
KEEP THIS IN MIND

Composition and resolution are the **same** rule run in opposite directions.

SET-UP

One force can replace many.

If a single pull would move a body exactly as several pulls do, we call it their **resultant**; the originals are its **components**.



- 1 Same effect**
The resultant produces exactly the motion the components together would.
- 2 Same body**
Only forces on the **one** object may be combined this way.
- 3 Either / or**
Use the resultant or the components — never both at once. It is a replacement, not an extra force.

THE EASY CASE

On one line, just add or subtract.

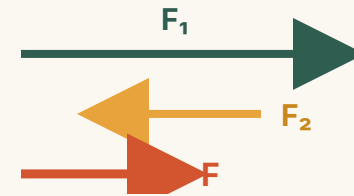
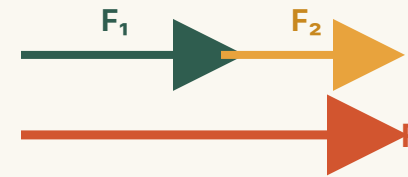
When two forces point along the same straight line, ordinary arithmetic works — direction decides the sign.

SAME DIRECTION

$F = F_1 + F_2$, pointing the same way.

OPPOSITE DIRECTIONS

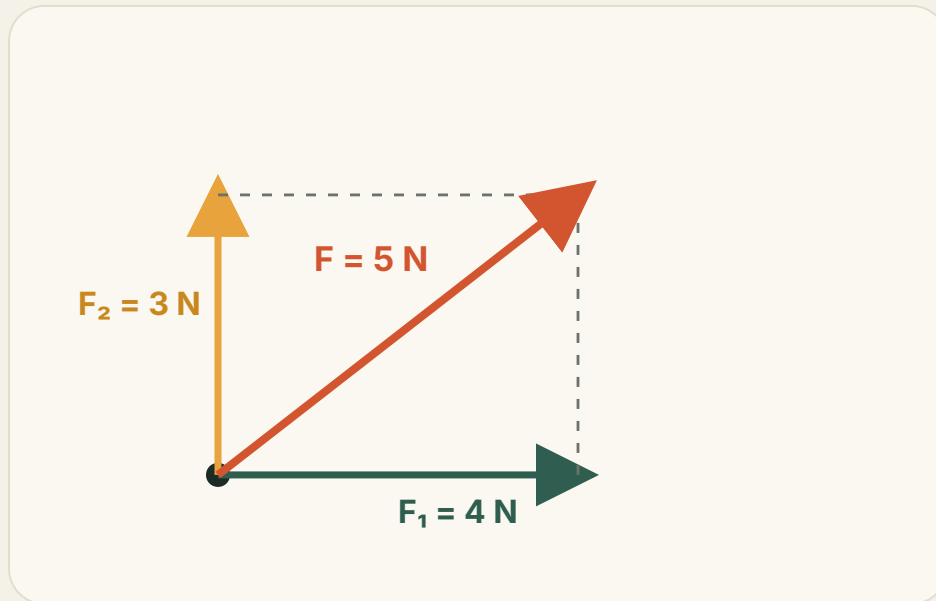
$F = |F_1 - F_2|$, pointing with the larger.



THE BIG IDEA

At an angle, the sum shrinks.

Pull a crate with 4 N east and 3 N north and you do not get 7 N.
The arrows must be joined head-to-tail.



Force has a **direction** as well as a size. Two forces partly pull across each other, so some of the effort is wasted sideways.

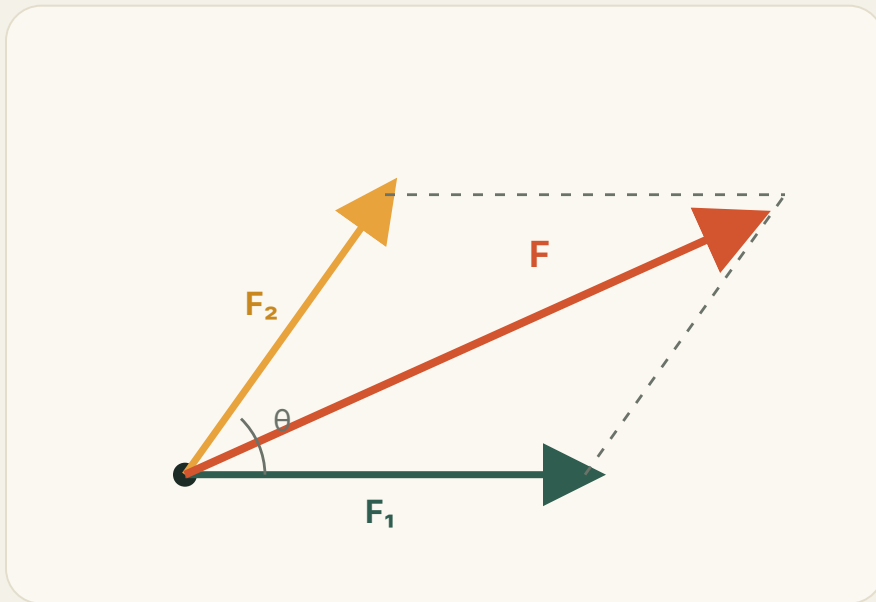
SO IN GENERAL

$$F \neq F_1 + F_2$$

THE RULE

The parallelogram rule

Draw the two forces from one point, complete the parallelogram, and the diagonal is the resultant.



$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

- θ the angle between the two forces
- F the diagonal of the parallelogram

TRIANGLE SHORTCUT

Or join them head-to-tail: the closing side, start to finish, is the same resultant.

WHAT THE RULE PROMISES

The resultant lives in a range.

Hold the two sizes fixed and swing the angle: the diagonal can only grow or shrink between two limits.

0° · Largest

Aligned, the resultant is the full sum, $F = F_1 + F_2$.

90° · Right angle

Perpendicular, use Pythagoras: $F = \sqrt{F_1^2 + F_2^2}$.

180° · Smallest

Opposed, it shrinks to the difference, $F = |F_1 - F_2|$.

↕ · In between

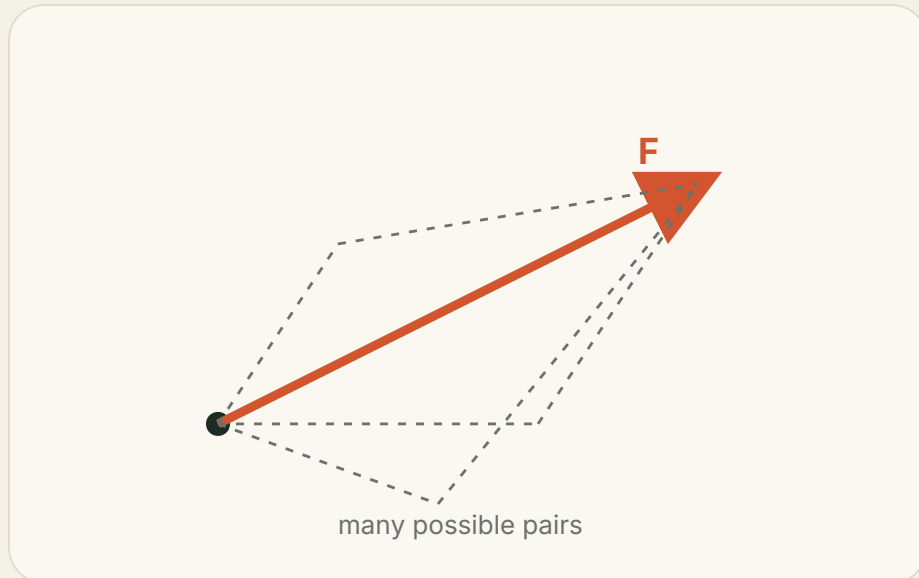
A wider angle always gives a smaller resultant.

THE WHOLE STORY IN ONE LINE

$$|F_1 - F_2| \leq F \leq F_1 + F_2$$

Resolving needs a rule to pin it down.

Splitting one force into two is the reverse of composition — but on its own it has infinitely many answers.



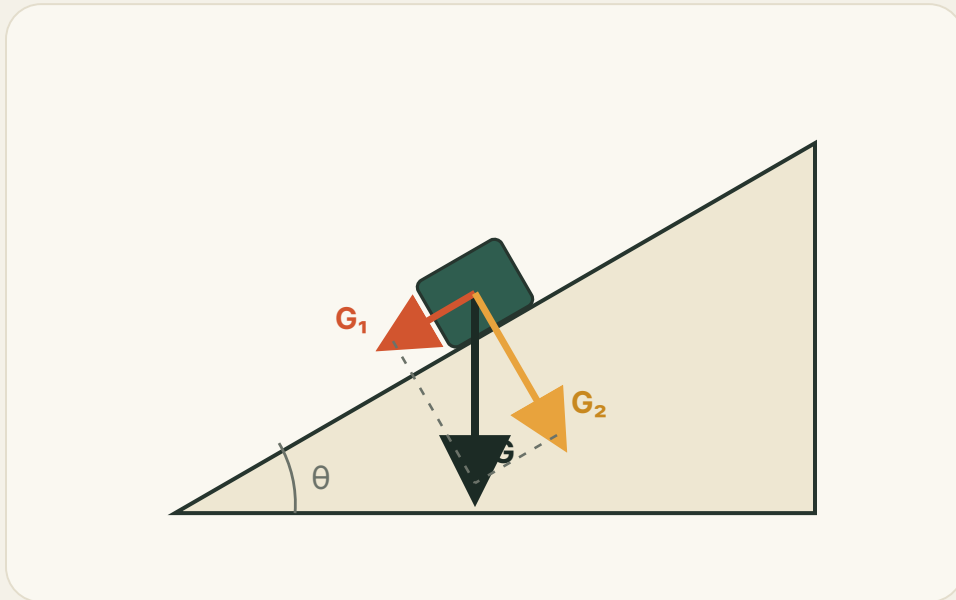
A single arrow is the diagonal of **endless** parallelograms. To get one answer you must add a constraint:

- 1 By effect**
Split along the two directions the force actually acts in.
- 2 By axes**
Split along chosen x and y directions — usually perpendicular.

RESOLVING BY EFFECT

A block on a slope shows the way.

Gravity does two jobs at once: it drags the block down the slope and presses it into the surface. Split it along those two effects.



DOWN THE SLOPE

$$G_1 = G \sin \theta$$

INTO THE SURFACE

$$G_2 = G \cos \theta$$

Steeper slope \rightarrow bigger $\sin \theta \rightarrow$ the block is pulled down harder.

WORKED EXAMPLE

Three and four make five.

A crate is pulled by 3 N north and 4 N east at right angles. Find the single equivalent pull.

1 Right angle, so use Pythagoras

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{4^2 + 3^2}$$

2 The magnitude

$$F = \sqrt{25} = 5 \text{ N} \text{ — less than the 7 N you might expect.}$$

3 The direction

$$\tan \alpha = \frac{3}{4} \Rightarrow \alpha \approx 37^\circ \text{ north of east.}$$

THE TAKE-AWAY

Forces combine like the sides of a **triangle**, not like a shopping bill. Geometry, not arithmetic, sets the total.

● SUMMARY

Force is a vector, so forces combine by the **parallelogram rule**: $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$, always between $|F_1 - F_2|$ and $F_1 + F_2$. Resolving is the same rule reversed – split a force where it does its work.