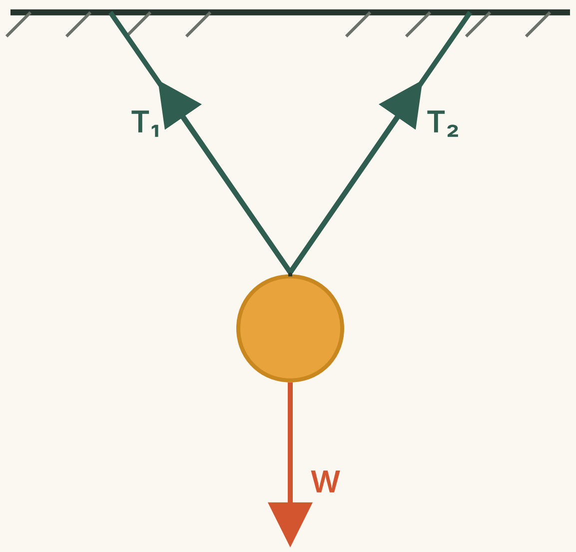
STATICS

Forces in Balance

When several pulls act at one point yet nothing moves, they must add up to exactly nothing.

ESSENTIAL QUESTION

What rule must a set of concurrent forces obey for an object to stay perfectly still — or glide at constant speed?



LESSON MAP

Reach the rule in four moves.

01 STATE

What "equilibrium" actually means — and when an object is in it.

02 CONDITION

The single demand: the resultant of all the forces is zero.

03 CASES

Two forces line up; three forces close a triangle.

04 SOLVE

Split into x and y, then read off the unknown forces.

KEEP THIS IN MIND

Concurrent forces are pulls whose lines of action meet at one point.

RECALL

No net force means no change in motion.

Newton's first law: with zero resultant force, an object stays at rest or keeps moving in a straight line at constant speed.

at rest: ΣF = 0

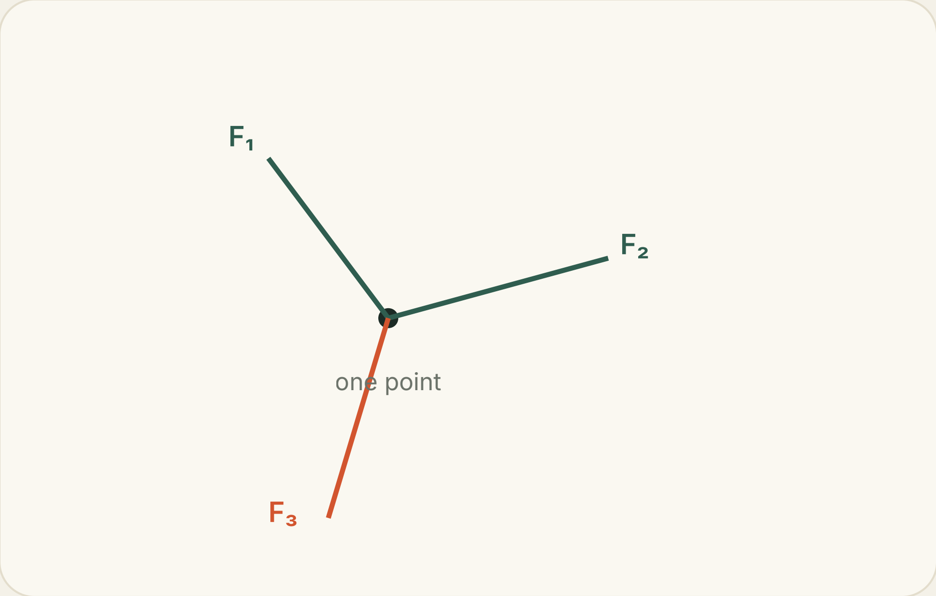
1. **Two equilibrium states**
   at rest, or in uniform straight-line motion — both have $a = 0$.
2. **Acceleration is the tell**
   zero acceleration is exactly what "balanced forces" produces.

So "balanced" is the **cause**; "no change in motion" is the **effect**.

THE BIG IDEA

Every pull is cancelled by the rest.

Slide the forces head-to-tail. If the object is in equilibrium, the chain comes right back to where it started.



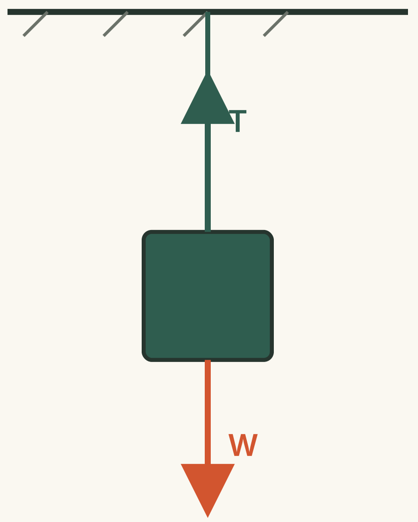
Concurrent forces share a point of action, so we can treat the object as a single dot and just add the arrows.

Equilibrium is the special case where that vector sum is the **zero vector** — no leftover push in any direction.

CASE ONE

Two forces: equal, opposite, in line.

The simplest balance. A hanging weight feels just the rope's tension up and gravity down.



THE TWO-FORCE RULE

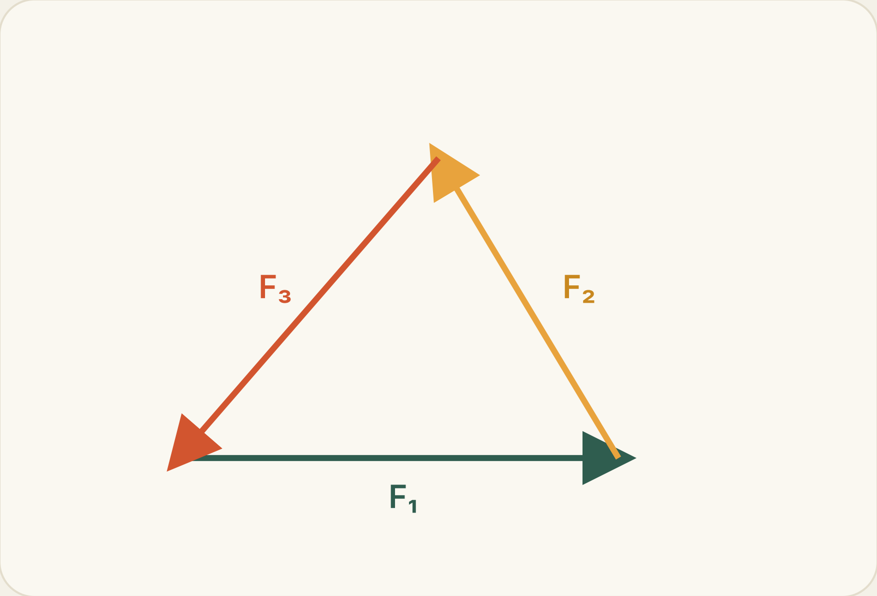
The forces must be **equal in size**, **opposite in direction**, and act **along the same line**.

$$T = W$$

CASE TWO

Three forces close a triangle.

Lay the three vectors head-to-tail. In equilibrium the last arrow lands exactly on the first — no gap.



Head-to-tail, the three forces form a closed loop.

1. **Any one balances the other two**
   one force equals the resultant of the rest, reversed.
2. **Their lines meet at a point**
   three non-parallel balanced forces are always concurrent.

The Condition for Equilibrium

$$\vec{F}_{\text{net}} = \sum \vec{F} = 0$$

A vector equation is really **two** scalar ones. Pick axes and demand balance on each:

$$\sum F_x = 0 \qquad \sum F_y = 0$$

ΣF	the vector sum of every force on the body
F_x	each force's component along the x-axis
F_y	each force's component along the y-axis
0	the zero vector — no resultant, so no acceleration

WHAT IT PROMISES

Four things equilibrium guarantees.

01 · Zero resultant

All the arrows, added tip-to-tail, return to the start.

02 · Static or uniform

Holds whether the body sits still or drifts at constant velocity.

03 · One balances the rest

Any single force equals the others' resultant, reversed.

04 · Concurrent lines

Three non-parallel balanced forces meet at a common point.

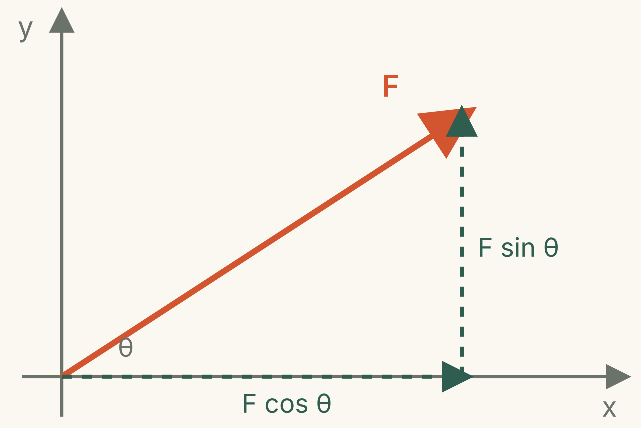
ONE PICTURE TO HOLD

Balanced on every axis: $\sum F_x = 0$ **and** $\sum F_y = 0$.

THE METHOD

Split each force onto two axes.

The reliable recipe: choose x and y, resolve every force, then set each column's sum to zero.



1. **Choose smart axes**
   often along and across a surface, so awkward forces vanish.
2. **Resolve every force**
   $F_x = F\cos\theta$, $F_y = F\sin\theta$.
3. **Set each sum to zero**
   two equations, solve for the unknowns.

WORKED EXAMPLE

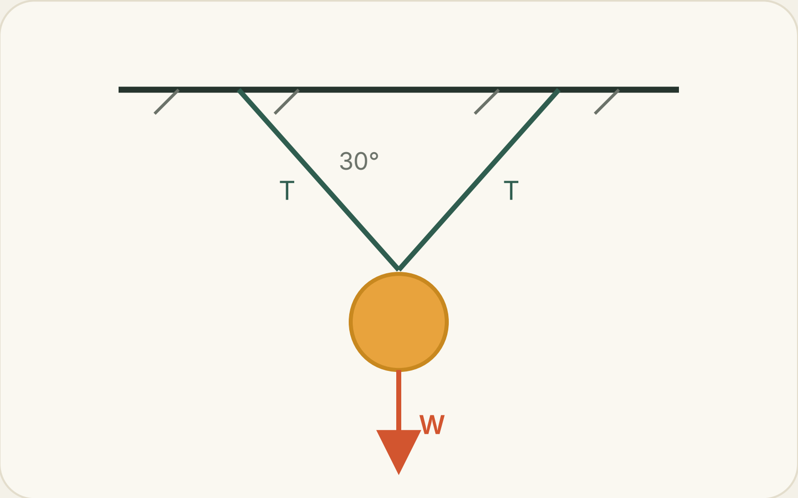
A lamp held by two slanted ropes.

A 20 N lamp hangs from a knot, each rope at 30° above the horizontal. Find the tension in each rope.

1. **Balance the vertical**
   two upward components hold the weight: $2T \sin 30° = W$.

2. **Solve for the tension**
   $T = \frac{W}{2 \sin 30°} = \frac{20}{2(0.5)} = 20 \text{ N}$.

3. **The horizontal looks after itself**
   by symmetry the two $T \cos 30°$ pulls cancel.



Forces that meet at a point hold an object in equilibrium only when their resultant is zero — $\sum F_x = 0$ and $\sum F_y = 0$. Resolve, balance each axis, and the unknown forces fall out.