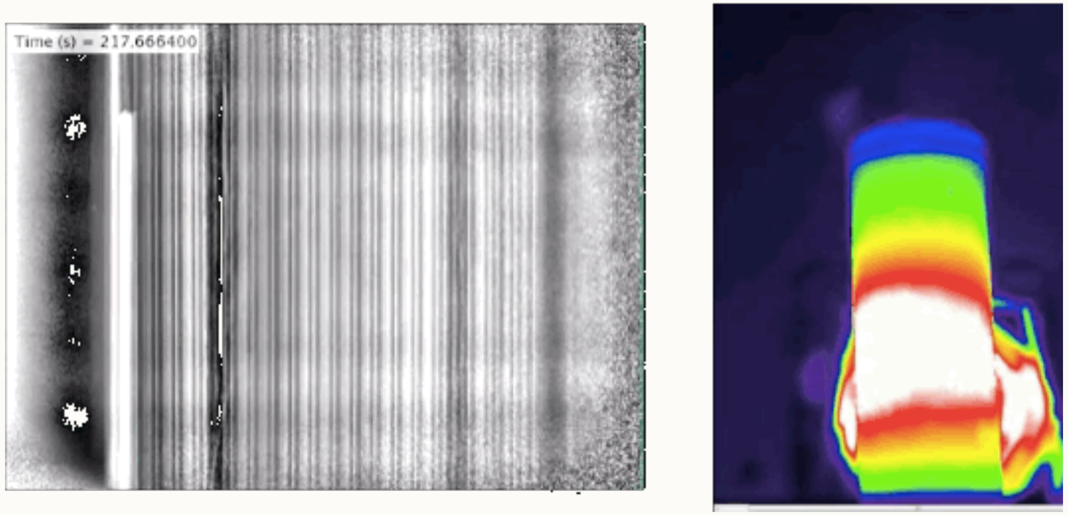
# Ohm's Law for Closed Circuits

Senior High Physics · Electric Circuits

OPENING QUESTION

# Why does a battery heat up?



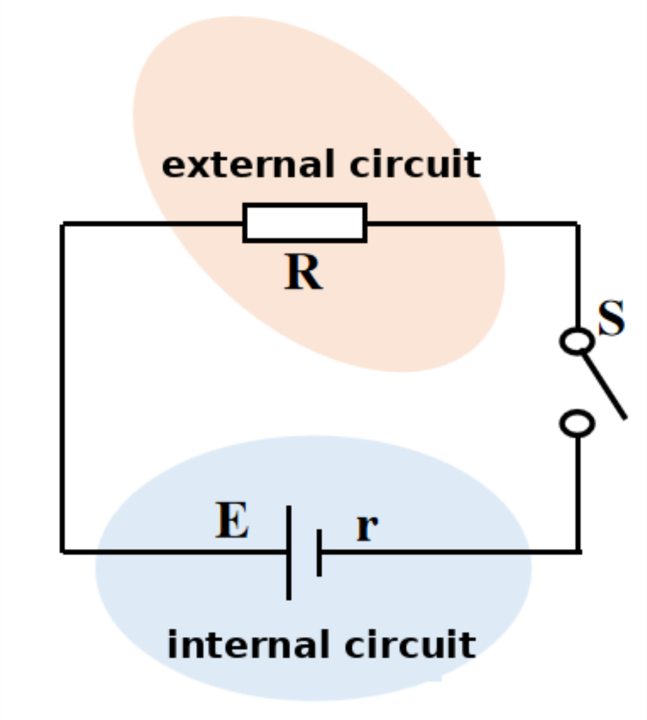

Thermal image of an overheating lithium-ion battery

… sometimes with dramatic consequences

> *And what is the expression for the heat it generates? Keep this question in mind — by the end of the lesson you will be able to answer it.*

THE BIG PICTURE

# A closed circuit has two parts




### EXTERNAL CIRCUIT

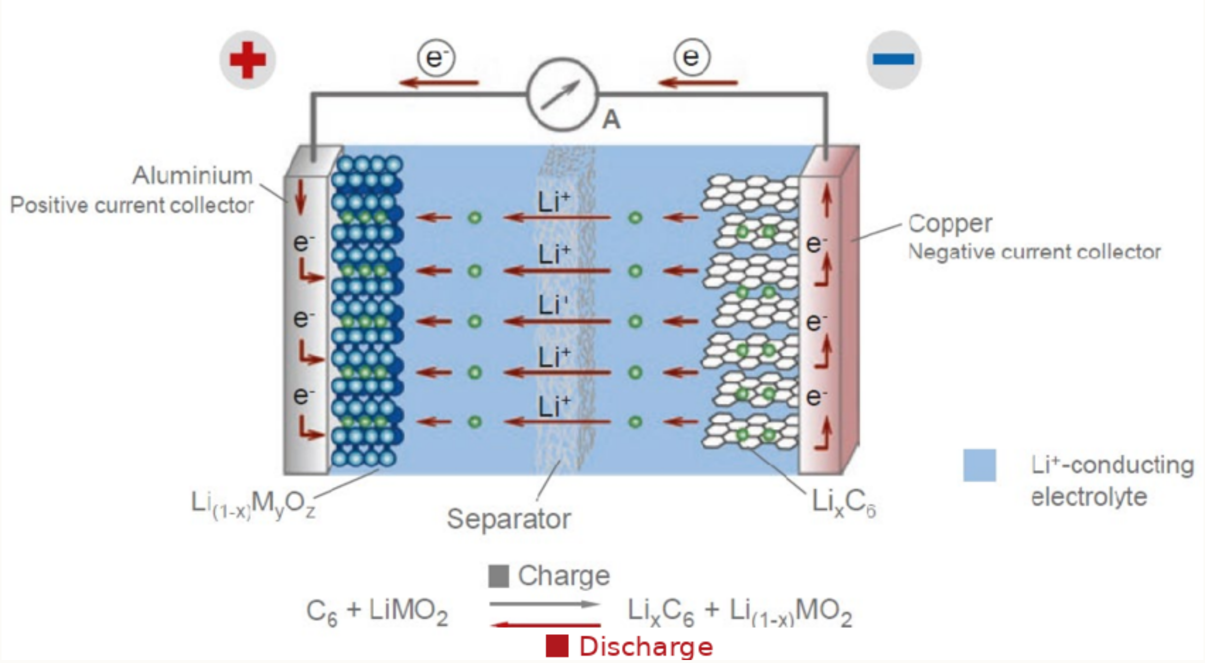
Everything outside the source: the resistance $R$, wires, switch — where the current does useful work.

### INTERNAL CIRCUIT

The inside of the source itself, with EMF $E$ and internal resistance $r$.

> *The current is the same everywhere in a series loop — including inside the battery.*

# What does the source actually do?




Ions and electrons on the move inside a lithium-ion cell during discharge

A source does not *create* charge. It **pumps** charge that is already there: positive charge is pushed from the low-potential terminal back up to the high-potential terminal, against the electric force.

> *Something inside the source must be doing work on the charges. What is it?*

NEW QUANTITY

# Electromotive force (EMF)

**PHYSICAL MEANING**

Measures how strongly a source converts other forms of energy into electric potential energy.

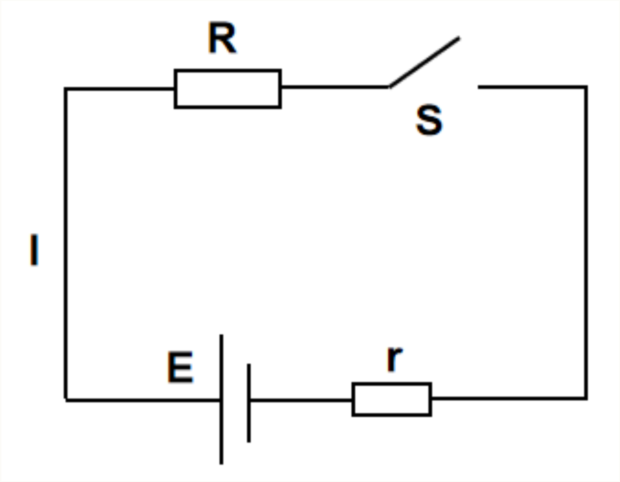
**DEFINITION**

The work done by non-electrostatic forces per unit charge moved through the source:

$$E = \frac{W}{q}$$

**UNIT**

The volt (V) — the same unit as voltage, but *not* the same quantity.

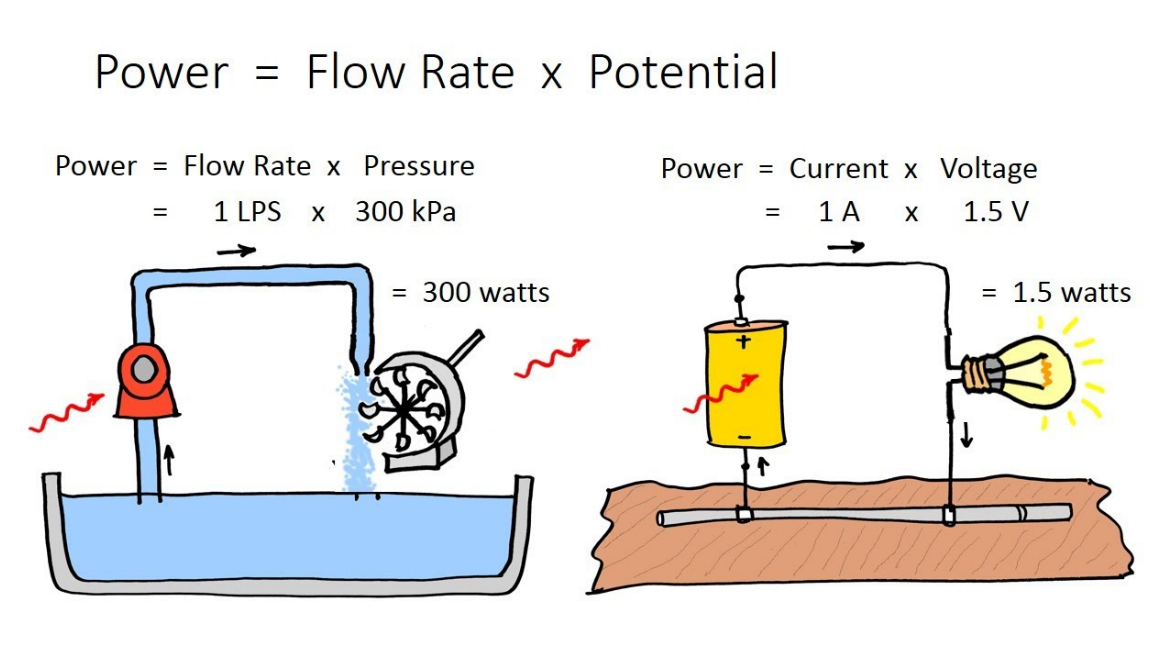



A source of EMF E and internal resistance r driving a current

> *A 1.5 V dry cell gives every coulomb of charge 1.5 J of energy as it passes through.*

ANALOGY

# EMF and voltage: the water model




The pump (source) lifts water; the waterwheel (resistor) extracts the energy

The **pump** raises water against gravity — like the source raising charge to a higher potential. The water flowing back down through the wheel is the current doing work in the external circuit.

> *EMF describes the pump's lifting ability; voltage describes the drop across each part of the path.*

## EXAMPLE 1 · UNDERSTANDING EMF

Which of the following statements about EMF is correct?

**A** The EMF of a source is proportional to the work done by non-electrostatic forces inside it, and inversely proportional to the charge that passes through

**B** EMF has the same unit as voltage, so EMF is just the voltage between the two terminals of a source

**C** The more work the non-electrostatic forces do, the greater the EMF

**D** The EMF is determined by the nature of the non-electrostatic forces inside the source ✓

**WHY**

$E = W/q$ is a *defining* equation — it does not mean $E \propto W$ or $E \propto 1/q$ (A, C ✗). EMF corresponds to work done by *non-electrostatic* forces; voltage corresponds to work done by electrostatic forces, so they are different quantities even though the unit is the same (B ✗).

## EXAMPLE 2 · COMPARING TWO BATTERIES

A lead–acid battery has an EMF of 2 V; a dry cell, 1.5 V. Which of the following is correct?

**A** The voltage between the terminals of the lead–acid battery is 2 V

**B** The dry cell can convert 1.5 J of chemical energy into electrical energy in 1 s

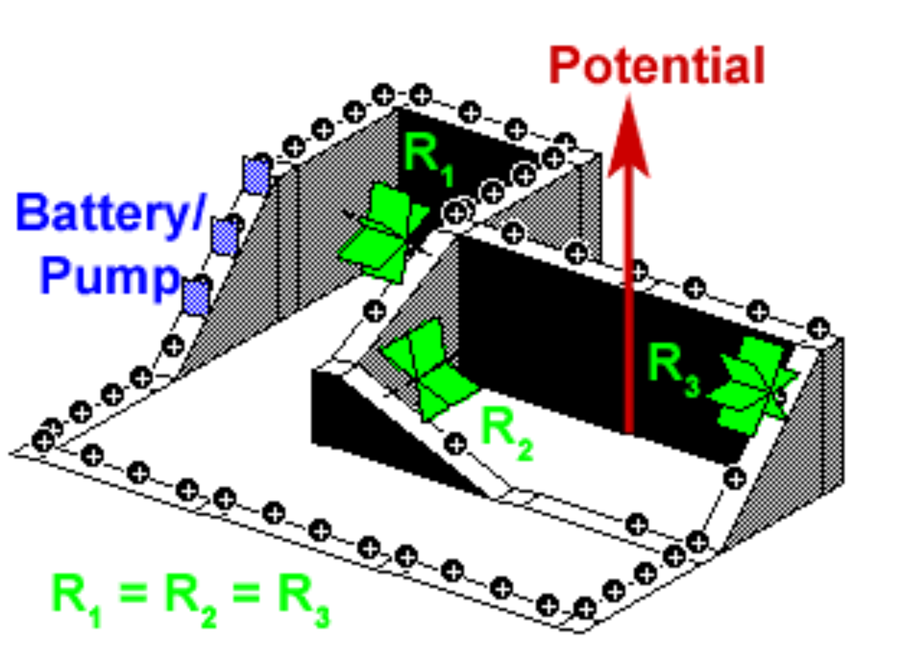
**C** The EMF of the lead–acid battery changes when it is connected to different circuits

**D** For the same charge passing through, the non-electrostatic forces do more work in the lead–acid battery than in the dry cell ✓

**WHY**

Terminal voltage is generally *less* than the EMF once current flows (A ✗). 1.5 V means 1.5 J *per coulomb*, not per second (B ✗). EMF is a property of the source itself — it does not depend on the circuit (C ✗). From $W = Eq$, a larger EMF means more work for the same charge (D ✓).

# How does the potential change around the loop?



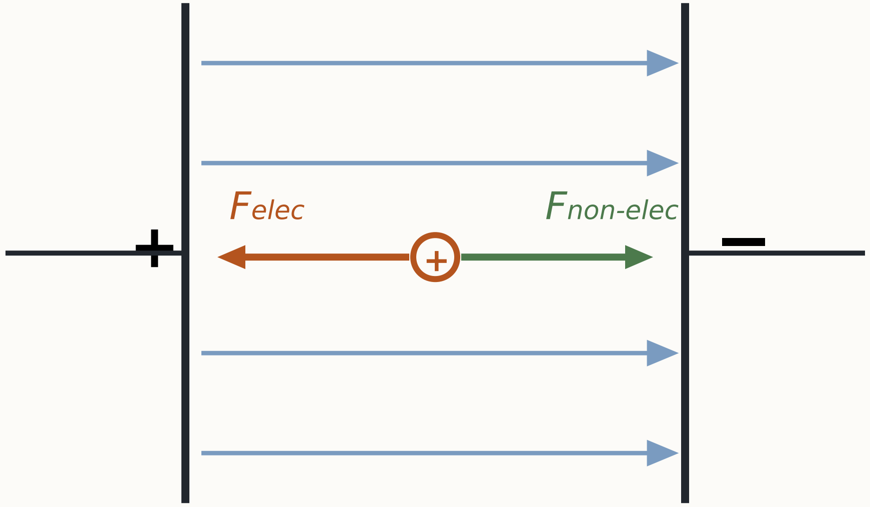

Potential around the circuit: a ramp down through the bulb, a sharp lift through the battery "pump"

Going around the external circuit, the potential **falls** step by step as the current passes each resistance.

Inside the source it must be **lifted back up** — otherwise the current could not keep flowing.

> *The battery is the "staircase" of the circuit: everything that comes down must be carried back up.*

# The non-electrostatic force




Inside the source, between the plates

Between the terminals the electric field pushes positive charge from + to – ($F_{elec}$).

To move charge the *other* way, the source provides a **non-electrostatic force** ($F_{non\text{-}elec}$) that does positive work on the charge, lifting its potential energy.

> *In a chemical battery this force comes from chemical reactions; in a generator, from electromagnetic induction.*

ANALOGY

# The water carrier

Carrying water uphill takes work — and somebody has to pay the energy bill

> *The carrier is the non-electrostatic force: he lifts the water (charge) back to the top against gravity (the electric force), spending his own energy (chemical energy) to do it.*

# Where the energy comes from

### CHEMICAL BATTERY

The non-electrostatic force is **chemical**: chemical energy → electric potential energy.

### GENERATOR

The non-electrostatic force is **electromagnetic**: mechanical energy → electric potential energy.

SUMMARY

# EMF, in one box

$$E = \frac{W}{q}$$

EMF measures how strongly a source converts other forms of energy into electric potential energy. Its unit is the **volt**.

The value of $E$ is decided by **the source itself** — it does not depend on the work $W$, on the charge $q$ moved, or on the external circuit.

## CHECK YOURSELF

**Q1.** Which statement about EMF is correct?

- **A** EMF is just voltage, and its unit is V
- **B** A source with a larger EMF must do more work
- **C** A source with a larger EMF must do work faster
- **D** The EMF equals the energy the source supplies for each unit of charge passing through the circuit ✓

**Q2.** The EMF of a source reflects its ability to convert other forms of energy into electrical energy. Therefore —

- **A** EMF is a kind of non-electrostatic force
- **B** A larger EMF means the source stores more electrical energy
- **C** The EMF reflects how much work the non-electrostatic forces can do per unit charge ✓
- **D** EMF is the voltage across the source in a closed circuit

PRACTICE

# Two batteries, two circuits

A lead–acid battery (EMF 2 V) and a dry cell (EMF 1.5 V) are connected to separate circuits, carrying currents of 0.1 A and 0.2 A respectively. Both circuits run for 20 s.

**How much chemical energy does each source consume? Which battery is better at converting chemical energy into electrical energy?**

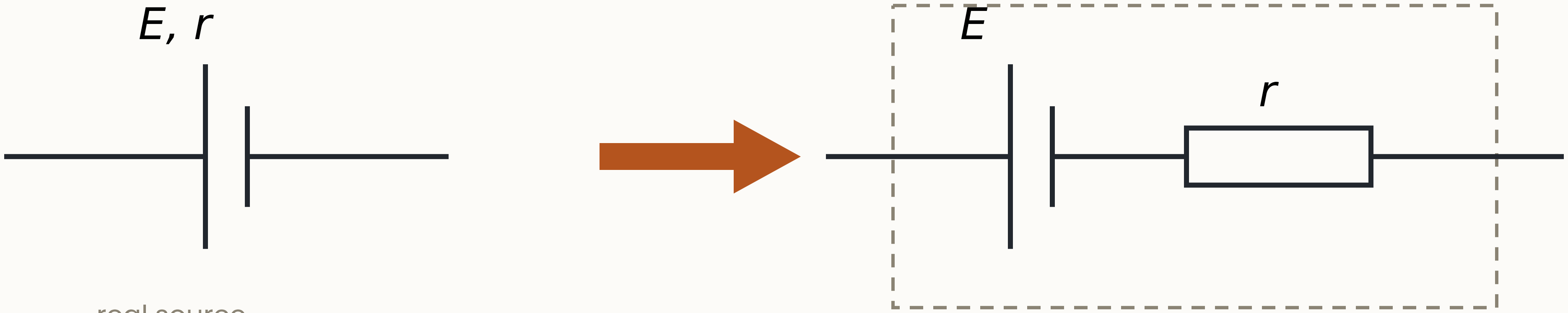
SOLUTION

$W = EIt$

Lead–acid: $W_1 = 2\ V \times 0.1\ A \times 20\ s = \mathbf{4\ J}$ | Dry cell: $W_2 = 1.5\ V \times 0.2\ A \times 20\ s = \mathbf{6\ J}$

The dry cell converts *more total energy* here — but "converting ability" means energy *per unit charge*, which is exactly the EMF. Since 2 V > 1.5 V, the **lead–acid battery** is the stronger converter.

# A real source = ideal source + resistor



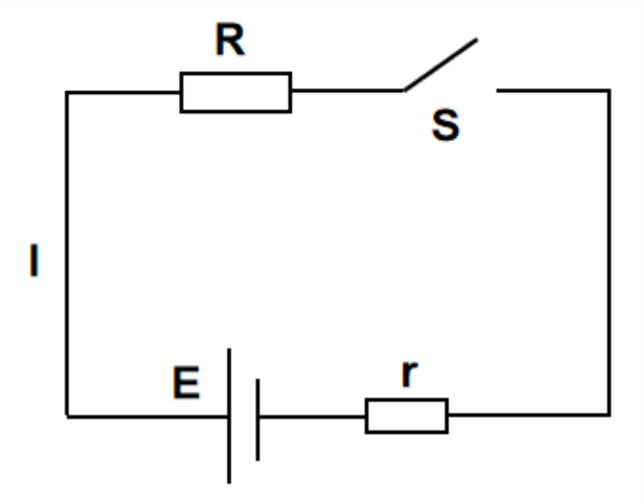

real source

ideal source (no resistance) in series with r

> *Electrolytes and electrodes resist current too. We pull that resistance out of the battery symbol and draw it as an ordinary resistor r — the **internal resistance**.*

KEY RELATIONSHIP

# Terminal voltage




EMF E, internal resistance r, external resistance R

Energy per unit charge from the source = energy dropped on $R$ + energy dropped on $r$:

$$E = U_{ext} + Ir$$

so the voltage actually available at the terminals is

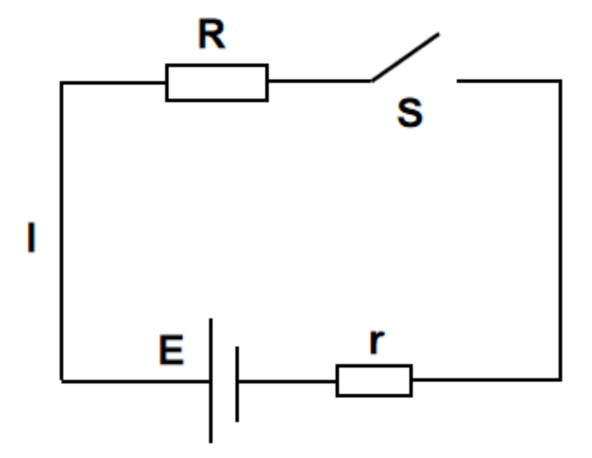
$$U_{ext} = E - Ir$$

$$I = \frac{E}{R + r}$$

> *The larger the current, the more voltage is "lost" inside the source. With no current, $U_{ext} = E$.*

EXAMPLE 3 · TERMINAL VOLTAGE

In the circuit shown, $R = 2.0\ \Omega$, the EMF of the source is $3.0\ V$ and its internal resistance is $r = 1.0\ \Omega$. After switch S is closed, what is the terminal voltage?



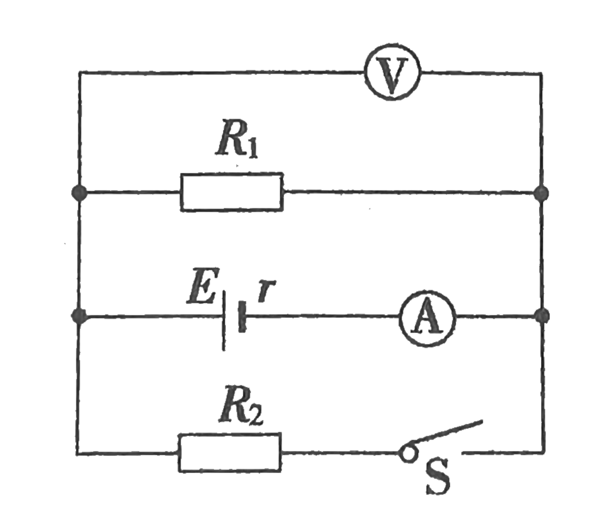

SOLUTION

$$I = \frac{E}{R + r} = \frac{3.0\ V}{2.0\ \Omega + 1.0\ \Omega} = 1.0\ A$$

$$U = E - Ir = 3.0\ V - 1.0\ A \times 1.0\ \Omega = \mathbf{2.0\ V}$$

### EXAMPLE 4 · FINDING E AND R FROM METER READINGS

When S is closed, the (ideal) voltmeter and ammeter read 1.6 V and 0.4 A. When S is opened, the readings change by 0.1 V and 0.1 A. Find the EMF and internal resistance of the source.




#### SOLUTION

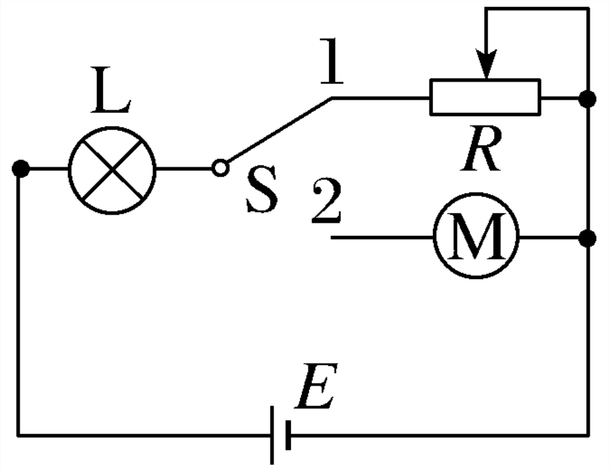
Opening S removes the $R_2$ branch, so the current falls and the terminal voltage rises: the two states are (1.6 V, 0.4 A) and (1.7 V, 0.3 A).

Apply $E = U + Ir$ to both:

$$r = \frac{\Delta U}{\Delta I} = \frac{0.1\ V}{0.1\ A} = \mathbf{1\,\Omega} \quad E = 1.6\ V + 0.4\ A \times 1\,\Omega = \mathbf{2.0\ V}$$

## EXAMPLE 5 · A CIRCUIT WITH A MOTOR

The source EMF is $E = 6\ V$. Bulb L is rated **"3 V, 0.9 W"**. With S at position 1 and the rheostat at $R = 8\ \Omega$, the bulb operates normally. With S at position 2, both the bulb and motor M work normally. The motor's coil resistance is $R_0 = 2\ \Omega$.




### (1) Find the internal resistance of the source.

**SOLUTION**

Normal operation: $R_L = \dfrac{U^2}{P} = \dfrac{(3\ V)^2}{0.9\ W} = 10\ \Omega,\quad I = \dfrac{P}{U} = 0.3\ A$

$E = I(R_L + r + R) \implies 6\ V = 0.3\ A \times (10\ \Omega + r + 8\ \Omega) \implies r = 2\ \Omega$

EXAMPLE 5 · CONTINUED

## (2) Find the motor's input power, output power and efficiency.

**SOLUTION**

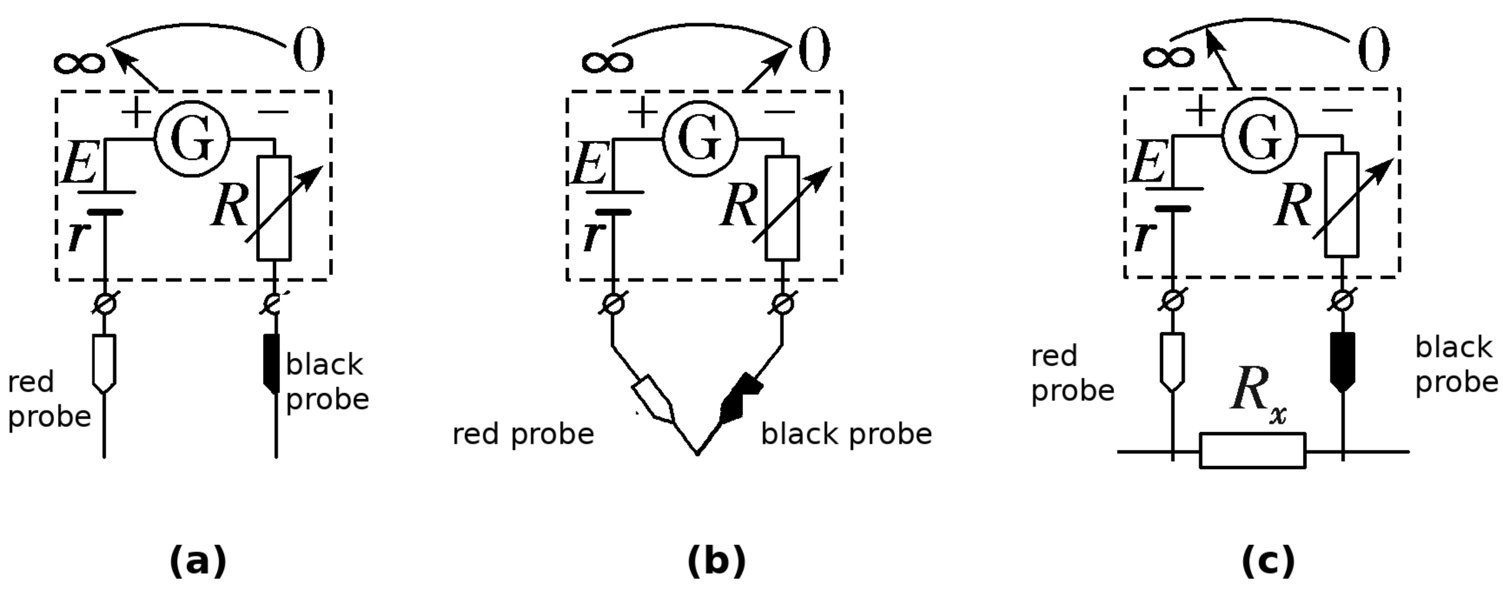
With S at 2 the bulb still works normally, so the current is unchanged: $I = 0.3\ A$. The motor's operating voltage:

$$U_M = E - I(R_L + r) = 6\ V - 0.3\ A \times 12\ \Omega = 2.4\ V$$

| QUANTITY | RELATION | VALUE |
|---|---|---|
| **Input power** | $P_{in} = U_M I$ | **0.72 W** |
| **Heating power** | $P_{heat} = I^2 R_0$ | 0.18 W |
| **Output power** | $P_{out} = P_{in} - P_{heat}$ | **0.54 W** |
| **Efficiency** | $\eta = P_{out} / P_{in}$ | **75%** |

> *A motor is not a pure resistor — Ohm's law applies to its coil resistance only, not to the motor as a whole.*

# The ohmmeter




(a) zero adjustment · (b) probes touched together · (c) measuring an unknown $R_x$

> *A battery (E, r), a galvanometer G and an adjustable resistor R in series — the measuring principle is exactly **Ohm's law for closed circuits**: a known EMF drives a current that depends on the unknown resistance.*

# From current to resistance

With an unknown $R_x$ across the probes:

$$I = \frac{E}{R + R_g + r + R_x} \implies R_x = \frac{E}{I} - (R + R_g + r)$$

Each $R_x$ gives exactly one current $I$ — so relabel the current scale with resistance values and read $R_x$ directly.

**Internal resistance:** $R_{int} = R + R_g + r$

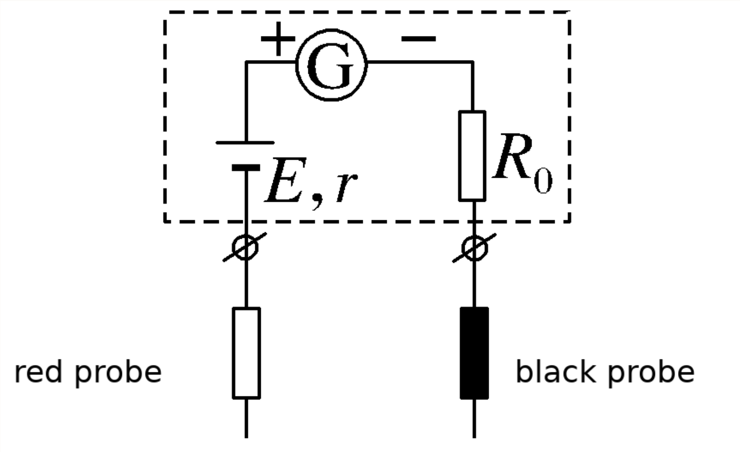
**Zero adjustment:** touch the probes, tune R until the pointer reads full scale ("0 Ω").

| MARK | CONDITION | POSITION |
|---|---|---|
| **"0 Ω"** | probes together, full deflection | far right ($I_g$) |
| **"∞"** | probes apart, no deflection | far left ($I = 0$) |
| **mid-scale** | $R_x = R_{int}$ | centre of dial |

> *The scale runs "backwards" (∞ on the left) and is non-uniform — most cramped at the high-resistance end.*

## WORKED EXAMPLE · OHMMETER SCALE

The meter's full-scale current is $I_g = 500\ \mu A$; the dry cell's EMF is 1.5 V. Fill in the resistance value for each current mark.




### SOLUTION

Zoreing (probes together, full scale): $R_{int} = \frac{E}{I_g} = \frac{1.5\ V}{500\ \mu A} = 3\ 000\ \Omega$

Measuring: $R_x = \frac{E}{I} - R_{int}$

| CURRENT MARK | 0 | 200 μA | 300 μA | 500 μA |
|---|---|---|---|---|
| RESISTANCE MARK | ∞ | **$4.5\times10^3\ \Omega$** | **$2\times10^3\ \Omega$** | **0** |

WORKED EXAMPLE · CONTINUED

**(2)** What is the total internal resistance of this ohmmeter? When the pointer deflects to ⅓ of full scale, what is the resistance being measured?

SOLUTION

From part (1), the total internal resistance is **3 000 Ω**.

At ⅓ of full scale: $I = \frac{I_g}{3} = \frac{500\ \mu A}{3}$

$$R_x = \frac{E}{I} - R_{int} = 3 \times 3\ 000\ \Omega - 3\ 000\ \Omega = \mathbf{6\ 000\ \Omega}$$

> *Handy rule: at 1/n of full scale,* $R_x = (n-1) \cdot R_{int}$.

# $E = U_{ext} + Ir$

One equation, the whole lesson: the source gives every unit of charge an energy $E$; part is delivered outside, part is lost inside. And the battery heats up because of that inside part — at a rate $P = I^2 r$.